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LAND SURVEYING

ON THE

Meridian & Perpendicular System

BY

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PREFACE.

OF late years, a feeling has been general among practical surveyors that a work upon Surveying by the Meridian and Perpendicular System would be acceptable, because, while forty or fifty years ago a number of books were published by eminent authorities, a larger knowledge of the means of applying this system practically has been afforded through the experience of colonial surveyors in recent times.

The popular method, in this country, is found by many to be inferior, while the application of the Trigonometrical or Geodetical System is very generally shunned, as being too intricate for the manipulation of ordinary work, although admitted by all to be the most accurate. Accordingly, the object of this work is to set forth a system, as based upon that practised in the Colonies, whereby small surveys may be executed in conformity with Trigonometrical principles, without increasing the labour to such an extent as to interfere with practicability.

EDINBURGH, *January* 1885.

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LAND SURVEYING ON THE MERIDIAN AND PERPENDICULAR SYSTEM.

I. THE CHAIN, AND HOW TO USE IT.

THERE are two kinds of chains used in this country, viz., the link chain and the foot chain; but all areas of land are measured by means of the former, causing it to become exclusively the instrument of the land surveyor; while the latter proves itself better adapted to the uses and requirements of the civil engineer and architect. Both chains, however, are divided into a hundred equal parts, each part being respectively a link or a foot in length. The link or Gunter's chain is 66 feet in length, or one-eightieth of a mile; therefore the length of each link in inches $= 66 + 12 \div 100 = 7{\cdot}92$. A link being therefore a decimal part of a chain, all area calculations may be readily worked out; thus 10 chains, that is to say 1000 links, multiplied by 1 chain or 100 links gives 100,000 square links: and by inserting the decimal point five places to the left, thus 1·00000, the quantity obtained is one acre. This, then, is the standard or imperial measure used by surveyors upon all occasions where an area is required.

In order to facilitate the reading of a distance, pieces of brass are fixed upon the chain every ten links apart. These are usually called tellers, and are constructed of various patterns so as to indicate readily, on inspection, their exact situation with reference to each end of the chain.

Chains of modern construction are made of steel instead of iron, which makes them lighter to carry or drag, and also easier to strain in a true horizontal position when measuring over rough or undulating ground; but the greatest advantage the steel chain has over the iron one is, that it is less liable to stretch or increase in length while in use. Actual practice shews that the amount of stretching in a chain is a serious matter, when it is known that a new chain may stretch ¼ of a link with ordinary usage in a day's work. This stretching is caused by the split rings which join the links, as well as the heads of the links themselves, yielding to the strain put upon them; accordingly, when the chain is laid upon the standard and tested, it becomes necessary to shorten it by removing a suitable number of rings at equally dispersed points throughout the chain. Through constant use, in a few days a chain reaches its maximum stretch, so that afterwards, with fair treatment, little correction is necessary. It is advisable, however, to test a chain on the standard very frequently, and at least once a day when in constant use upon a large survey.

Within the last few years the steel band has come into use among surveyors—especially in the colonies, and has met with great success. This is simply a steel band about ¾ of an inch in breadth, and having ordinary chain handles fixed on with swivels at each end; it is carefully tested by the manufacturers to 66 feet in length. Each link is marked by a small brass rivet, and at every tenth, a brass plate is fixed and marked in the same way as the teller of an ordinary chain. The band is wound upon a metal frame when not in use, and being only about one half the weight of a common chain, it is easier to drag, and does not collect dirt in heavy ground. These in themselves are recommendations, but the great advantage of the steel band is, that it may always be relied upon with regard to adjustment, and being light as well as rigid, it may be strained to the horizontal with ease and security. Surveyors who use this form of chain speak highly of its results; considering that in its use the temperature correction may be cast aside; and that it is of all chains the one most suited to the requirements of a survey, either with respect to the measurement of a base, or the ordinary traverse. Before explaining how to use the chain, it should be understood how to lay down a standard of adjustment.

Proceed to a government standard, and correct the chain to it, holding the outside of one handle to one mark, and adjusting till the other point coincides with the inside of the other handle. Most surveyors take the standard of measurement to the outside of both handles, and in chaining make the arrows touch the outside of the handles also; this system is doubtless convenient, but it is not theoretically correct. The chainage should be taken from centre to centre of the arrows; consequently by the above method, for every chain laid, the thickness of an arrow is lost. Neither can the correct result be given by causing one of the arrows to be put inside the handle of a chain thus adjusted; because the thickness of one handle would be added to one half the thickness of the arrow, which would be an opposite and greater error. The only accurate way, therefore, is, as already stated, to cause the chain to be adjusted to 66 feet from outside to inside of handles; and in chaining, to instruct one of the chainmen to hold the outside of his handle to the arrow, and the other to place his arrow against the inside of his handle. The following Fig. 1 will illustrate what is meant.

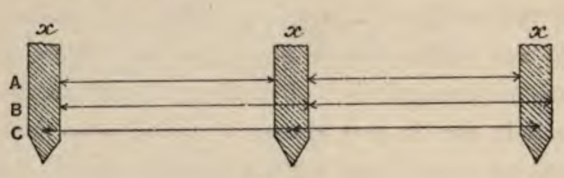


Fig. 1.

Let *x.x.x.* represent three arrows, and *A.B.C.* three cases.

In case *A.* the chain is tested from outside to outside, and the arrows placed in the same position; it may be at once seen that in two chains the thickness of two arrows is lost. In case *B.*, however, the chain is tested from outside to inside, and the arrows similarly placed in chaining, it will be seen at a glance that this gives a correct result, as shewn in case *C.*

Having made the adjustments as explained; in order to lay down a private standard, select a flat piece of ground not likely to be interfered with by traffic, and at one extremity of the proposed standard drive a peg into the ground. This peg should be about 3 feet long, and 3 inches square on the head, driven so as to leave the top on a level with the surface. Into the head of this peg drive a strong nail, leaving about $\frac{3}{4}$ of an inch of it projecting vertically from the top of the peg; over this hang one of the chain handles, and stretch the 66 feet upon the site of the proposed standard. Look over and examine the chain to see that there are no cranks or twists upon it; and then mark upon the ground the position of the outside of the other handle, and drive in a similar peg level with the surface. Strain the chain with the usual tension given as in measuring, and mark on the head of the peg by a fine cut with a knife, the position of the outside of the handle, and complete the operation by driving a small tack into the centre of the cut-mark, which will be the correct standard of measurement. It will be seen that by means of the nail in the head of the first peg, only one person is required to test a chain, and that there is no chance of the chain being allowed to "give."

Having thus obtained a correctly adjusted chain, the next matter under consideration is how to use it to its best advantage in the measurement of required distances; and as the chaining of a base line necessitates the greatest amount of accuracy, a description of the manner of accomplishing this important task may be taken as an example for the treatment of chainage by chainmen.

Two points having been fixed upon as the extremities of the base, and several intermediate points ranged and marked by ranging rods to lay out the course in a perfectly straight line, the chainmen proceed to one extremity of the base, and extend the chain in the direction of the other. If it is an ordinary chain that is used, a thorough inspection must be made, to see that there are no cranks upon it, and that all the tellers are in their proper places; should it be a steel band it is obvious that this scrutiny will be unnecessary.

Now, in consulting most authorities on land surveying, it will be found that in their opinion the more careful or responsible chainman should follow or *drive* the chain; while, on the other hand, a large number of practical land-surveyors consider that no exception should be made to the rule of putting the best man first, for the simple reason that it is more difficult to stick in an arrow accurately, than to lift it. It may be urged that the *driver* has to keep the *leader* in a true line: but a sharp leader will always line himself after the first chain is laid, and the more easily and accurately will he do so after passing one of the intermediates. This fact need not be urged further, as a very little experience with a bad leader will go a long way to convince a surveyor as to the truth of these arguments.

Returning again to the chaining of the base, instruct the best chainman or leader to count his arrows to see that he has *ten* before starting, and let him select one for use. The *following* end of the chain being held hard to the exact centre of the starting pole by the driver, the leader must hold the arrow vertically against the inside centre of the handle, with only about two inches of it projecting below the handle, so that it cannot be stuck into the ground otherwise than vertically.

The leader should completely face the driver, when the latter, by a motion of the head (not with the hand, as he must have both hands to hold back the chain), signals the chain into line, and at once cries " mark." The leader having straightened out the chain, and strained it with the same amount of pull as in adjusting, should then instantly cause his arrow to catch the ground, and holding the handle with one hand, push in the arrow with the other, at the same time calling out " one." The chain is then dragged onwards clear of the arrow, which the driver by stepping 22 yards can have no difficulty in finding, the leader getting another arrow ready in the inside of his handle so as to cause no unnecessary delay. The driver arriving at the arrow already sticking in, calls out to the leader " stop," and standing directly over the arrow, again signals the chain into the straight line. The leader, however, now finds that he can place himself in line by means of the starting-pole behind the driver, so he shakes the chain into position by vertical jerks, and prepares to mark as before. The driver, meanwhile holding his handle tight against the arrow with the points of the fingers of both hands, and supporting the arrow or holding against the strain with his thumbs, again calls out " mark," and on (but not before) receiving the answer "two," he releases one hand from the chain, and lifts the arrow. The same process goes on until the tenth arrow is set and reached by the driver. The chain is then set in the same manner as before, but upon the driver calling out "change," the leader carefully marks upon the ground the position of the inside of his handle, either by placing his foot firmly upon the handle itself, or by sticking in a pocket-knife or twig in the ground. The driver then takes forward the arrows, and after carefully counting them, hands them to the leader, saying "count ten;" this the leader does for his own satisfaction, and making a note of the change in his book, sticks in an arrow, calls out "eleven," and drags the chain onwards.

Thus the chaining proceeds, the arrows being always counted at the changes, and the number of changes noted in the leader's book, until the last chain is laid down, and its arrow stuck in. The leader then sums up the number of changes as noted, which represent ten chains each, and adds the odd number of chains as indicated by the number of arrows in the driver's hand, including the one sticking nearest the termination of the line. Then, checking the result by the number of his own arrows to find if he has the balance of *ten*, the odd links which have to be added are measured, either by pulling the chain on again so as to read from the arrow to the centre of the rod, or by holding the handle to the centre, and reading back to the arrow. There can be no preference to either way as far as the result is concerned, but it is advisable to adopt one of the two ways and keep to it as much as possible, because chainmen are liable to be easily confused. Before noting down the odd links, the driver should read and state the result also, and upon no account must the arrow be lifted, or the others returned to either party, until the measurement is actually noted down, and mutually agreed upon. During the operation, if any arrow stuck into the ground should be accidentally lost, or dragged out by the chain, the line must be commenced afresh from the beginning; as the surveyor must not only have his results accurate, but must be certain of their accuracy; and if chainmen are made to understand this, they will be careful to avoid these things.

For simplicity in calculation, it is usual to suppose the link to be divided into 10 parts, these of course, being approximated by the eye. Supposing the leader to have noted seven changes, and while he had two arrows in his hand, the driver had seven, and counting the last arrow put in makes eight altogether; and suppose the odd links from the eighth to the centre of the rod, to read 32½ links,—the total length of the base, calling it *A.B.* will read 7832·5.

As will be pointed out afterwards, it is necessary to chain *the base line* at least three times, taking the average as the result; the ordinary traverse lines however, only require to be chained once, unless in exceptional circumstances, or when they happen to be over very rough ground.

In the example already given of the method of chaining, it was assumed that the ground lying between *A* and *B* was perfectly level, but unfortunately declivities and rough ground are more commonly met with in field work: it is thus necessary to have some standard to which to reduce the results, otherwise the conclusions arrived at by the measurement would be conflicting and irreconcileable on account of the different gradients met with; consequently, the survey would not close. In explanation of what is here meant, take an example, as in the case of Azimuth.

Two surveyors have instructions to measure the exact distance, as the crow flies, from a certain point in one town, to a point in another, otherwise than by chaining the straight line, and each by a different route. The first surveyor traverses along the turnpike road, while the second traverses an outlying road, taking a much greater circuit. It is evident that, were the surveyors to furnish simply their respective chainages as their results, the answers would not agree at all, even although each surveyor had done his chaining with perfect accuracy. But let these chainages be reduced to the straight line by means of trigonometrical calculation, it will be found that the required distance between the points will be precisely the same, although the routes taken have been divergent. Applying this principle to the vertical plane, if two separate traverses were taken from one point to another, by routes differing in declivity, it would be impossible to reconcile these distances unless by reducing them to a normal standard :—that standard is the horizontal distance. In surveying, the application of this principle is an absolute necessity. Before attempting to explain the method of measurement and reduction to the horizontal, it will be better to consider what effect this rule has upon the measurement of lineal distances, or of land generally.

At first sight it appears hardly correct to make the base of a sugar loaf stand as equal to the superficial area of the whole; but it may be shewn that, looking at this proposition from a surveyor's point of view, in some cases it may be right, while in others wrong, to do so.

Take for example the case of a field having steep slopes, represented by the section *A.B.C.*, Fig. 2; and let each dotted line above the section represent growing crop, such as wheat, oats, or barley. As these crops naturally grow in a vertical position, by drawing vertical lines through each downwards upon the base *A.C.*, it is clear that, as every line so drawn rests

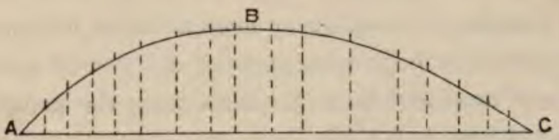


Fig. 2.

upon the base, and between the points *A.C.*, however many more were added, the same result would follow. Consequently, the line *A.C.* is capable of having the same capacity in this respect as the surface line *A.B.C.*, which plainly indicates that as much crop of this nature can be grown upon *A.C.*, as upon *A.B.C.* Accordingly it would be perfectly consistent to cause the measurement *A.C.* to represent *A.B.C.*

Again, supposing a drain had been cut along the surface *A.B.C.*, it is obvious that it, having no relation to the vertical, must consequently be longer than *A.C.*, so that in this case it would be incorrect to make the measurement *A.C.* stand for *A.B.C.* Take one other example, viz., as in the erection of a post and rail fence upon the surface line *A.B.C.* From the foregoing deductions it would appear, that while on the one hand the same number of posts would be required as in the fencing of *A.C.*, on the other, a shorter rail would be used for *A.C.;* but in order to keep the fence the same height, the posts would also require to be shorter. A surveyor should take these facts into consideration, and in measuring, whether it be crops, drains, or otherwise, have an intelligent reason to give if required, because this is a matter of very popular speculation amongst people not conversant with these facts.

There are two methods of determining the true horizontal distance of a measurement. The one is by what is usually termed plumbing; the other, or more accurate, is by ascertaining the altitude or depression angle of the ground upon which the chain is used, and from this, calculating the percentage to be deducted from each chain.

In the case of plumbing—when the chaining is proceeding from the top of a gradient downwards, the driver holds his handle hard to the bottom of his arrow, while the leader either strains the whole chain out to the horizontal position, or, if the ground is too steep to admit of this, selects any point upon the chain nearer the driver, at which he can comfortably do so. Having satisfied himself that he is holding the chain to the proper height, he exerts an extra pull in order to free the chain from the curvature caused by its own weight, and having suspended a heavy sharp-pointed plumb-bob from the point selected, lets it drop by allowing the string to slip through his hand, upon receiving the word "mark" from the driver. If the point selected is the handle of the chain, the string of the plumb-bob must be made to coincide with the handle; but if the chain is *broken,* which means that the whole length could not be taken at once, then, after fixing the point upon the ground with the plumb-bob, it must be temporarily marked, say with the blade of a knife, and the exact spot upon the chain handed over to the driver, and applied by him to the blade of the knife for a fresh *break* or plumb.

When chaining up a slope, the plumbing is of course done by the driver, and the leader marks instantly upon receiving a signal from the driver that the point of the plumb-bob coincides with the point previously measured to. It will be found better to chain down a slope where much plumbing is required, because in so doing the driver has always a fixed point to hold to, whereas in chaining upwards, the point is only indicated through the assistance of the plumb-bob line, which does not easily admit of precision. Great care is necessary in order to judge properly as to when the chain is in a true horizontal position; but this may be tested by holding the chain a little higher or lower, and observing the alteration in the lateral position of the plumb-bob, bearing in mind that the nearer the chain to the true horizontal, the further will be the correct point from the beginning of the chain line.

Manifestly, upon steep, undulating, or rough ground the operation of plumbing requires an immense amount of care and accuracy. Indeed, it may be said that chainmen require to exert a great deal of activity and judgment in order to supply anything approaching satisfactory results. The time occupied in plumbing one single chain properly, is often as much as required in the laying of five or six chains upon level ground; and further, the strain necessarily exerted in plumbing with a common chain, causes it to stretch often to such a serious extent as to induce errors to creep into the work, so that the use of the steel band is greatly preferable in plumbing. The better method, however, in arriving at the horizontal distance of chainage, is by means of the angle of altitude or depression which the surface of the ground makes with the level or horizontal line.

This angle may be obtained by setting up a theodolite, levelling it, and observing the vertical angle of a point at the other extremity of the gradient, at the same height from the ground as the centre of the telescope; but as the theodolite takes some time to adjust, a small pocket instrument called the clinometer (of which a description will afterwards be given) is now generally used, and found to give good results.

The following example, Fig. 3, is intended to represent a section of ground. It is required to chain from *A.* to *B.*, reducing the same to the horizontal distance *C.D.*

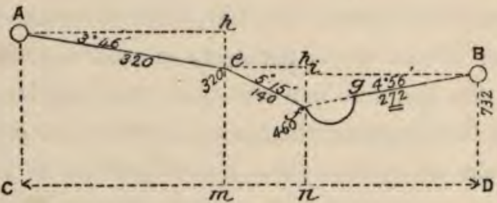


Fig. 3.

Let the depression angle of part of the line, as *A.e.*, be 3° 46′ by the instrument. The chaining then begins at *A.*, and goes onwards to *e.* without plumbing, but as there is a change of gradient at that point, the distance *A.e.* = 320 must be noted, and the next angle of gradient *e. f.* observed—which is shewn as 5° 15′. The chaining is again continued upon the slope as formerly until point *f.* is reached, and the distance 460 noted down. Let it now be supposed that the ground between *f.* and *B.*, with the exception of the portion *f.g.* is in one gradient; but as the distance *f.* to *g.* can be spanned by the chain, no account need be taken of it, except that care is required to keep the chain in the same gradient, and free from the curvature caused by its own weight; the last angle and total distance are then carefully noted.

Now, let the dotted lines upon the Fig. represent the true horizontal and vertical. From this it will be seen, that each chained gradient is the hypotenuse of a right-angled triangle; and as in each case one of the angles has been ascertained, the horizontal distance may be readily calculated by the rules of trigonometry, namely, *A.n.*, *e.k.*, *l.B.*; the sum of these will manifestly be equal to *C.D.* :—the distance required. The following are the calculations :—

$$\begin{aligned}
C.m \text{ or } A.h &= \log \text{Cos } 3^\circ\, 46' = 9 \cdot 9990608 \\
\log\ & \phantom{\text{Cos } 3^\circ\,} 320 = \underline{2 \cdot 5051500} \\
& C.m = 12 \cdot 5042108 = 319 \cdot 3 = \textit{ans.} \\[1ex]
m.n \text{ or } e.K &= \log \text{Cos } 5^\circ\, 15' = 9 \cdot 9981743 \\
\log\ & \phantom{\text{Cos } 5^\circ\,} 140 = \underline{2 \cdot 1461280} \\
& m.n = 12 \cdot 1443023 = 139 \cdot 4 = \textit{ans.} \\[1ex]
n.D \text{ or } l.B &= \log \text{Cos } 4^\circ\, 56' = 9 \cdot 9983881 \\
\log\ & \phantom{\text{Cos } 4^\circ\,} 272 = \underline{2 \cdot 4345689} \\
& n.D = 12 \cdot 4329570 = \underline{271 \cdot 0} = \textit{ans.} \\
& \text{Total} \quad = \underline{\underline{729 \cdot 7}} = C.D
\end{aligned}$$

These angles must be noted down, each in conjunction with its own distance, and the result calculated at leisure. As the chainage error is always, or ought to be, greater than the true distance, it is a practice amongst surveyors to read the chain a little less than it actually shews the length of the line to be; this is a habit which should be universally condemned. The distances ought to be taken and booked exactly as they read upon the chain; it is surely time enough to make deductions when it has been proved by calculation that deductions are necessary, and their exact amount ascertained. When chainmen notice that the surveyor, instead of accepting their results, proceeds to make alterations, it has a tendency to cause them to be less careful with their work, and removes that sense of responsibility which good chainmen ought to have for the correctness of their returns. Bad chaining is a source of great annoyance and risk to a surveyor, and in order to correct it, and find out the exact nature and position of the mistake, it is often necessary to do the chaining over again between the affected points. More depends upon accurate chaining than upon any other operation in surveying, as the angular checks, if at all carefully observed, are in most cases sufficient to demonstrate their own accuracy. In order, however, to have the work truly correct, both chain and instrument must be handled with care and judgment.

II. AUXILIARY INSTRUMENTS USED IN THE SURVEY.

The Optical Square.

For the working of a survey in the field, it is found very necessary to have some sort of instrument at disposal, by which a perpendicular may readily be set off to or from a given point; within the last twenty years or so, a small pocket instrument called the optical square has been in general use for this purpose.

This little instrument has almost entirely superseded the clumsy cross staff, which used to be the necessary burden of surveyors fifty years ago; and the complete handiness with which it can be carried and applied, as well as the ease with which it may be adjusted, or detected when not out of adjustment, has rightly made it the surveyor's pocket companion.

Before explaining its use, it may be well to examine the theoretical principles of its construction, as represented by Figs. 4 and 5.

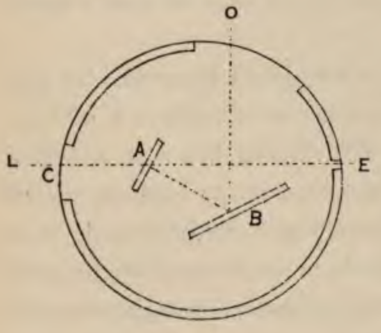


Fig. 4.

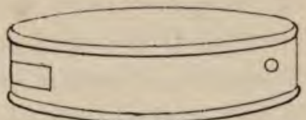


Fig. 5.

It consists in the first place, of a small circular box containing two mirrors, *A.* and *B.* The smaller of the two, viz., *A.*, is, however, only partially silvered, so that the top of it is transparent. Both glasses are fixed by adjusting screws in positions relative to the optical principle of the equality of the angles of incidence and reflection; which in this case is 45°.

Thus, when the instrument is held in the hand immediately over the spot where the perpendicular is desired, and the eye of the observer applied to the small hole at *E.*, looking through the transparent portion of the mirror *A.*, and through the opening *C.*, by turning the instrument a little if necessary, he will be able to see straight before him in the line *E.L.*, towards the fixed point. Then, upon looking steadily at the reflecting portion of the same mirror, a pole or ranging rod may be moved by an assistant, until at *O.* it appears exactly to coincide with the fixed point; then the point *O.* is at right angles to the line *E.L.*, at the observer's standpoint.

To find where a perpendicular from a distant object would strike a line, walk along the line until it appears to the ordinary senses nearly perpendicular. Then, applying the instrument to the eye as before, move slowly in the line, backwards or forwards, until the object is seen in the silvered part of the glass to coincide with the line as seen forward through the transparent part.

The instrument may be tested very easily by reversion; that is, by sighting upon the perpendicular already set off, and observing if the remainder of the original line still coincides. If an error is found to exist, the mirrors may be easily moved by means of the adjusting screws which hold them in their frames. The optical square should never be used where great accuracy is required; but it will be found thoroughly suitable for the setting of offsets in a survey, or the ranging of lines at right angles in land measuring.

THE CLINOMETER.

This, like the optical square, is also a pocket instrument, but its use is for the determining of the angle of altitude or depression which a gradient bears with relation to the horizontal, or level line; so that it is an instrument suited to the requirements of the chainmen. Fig. 6 represents a drawing of the most improved pattern of this instrument. It consists of a brass box or telescope *a.a.*, having an eye piece at one end *b.*, and a half-silvered glass near the other *c.* Underneath is fixed a divided arc *g.g.*, upon which a vernier *e.* moves. A small sensitive spirit level *d.* is fixed upon a moveable axis, which is turned by means of the side wheel *f.* The vernier is also attached to the moveable axis, so that it and the spirit level move together. Immediately above the centre of the spirit level, and in front of the mirror *c*, a hole is cut in the telescope tube, and as the mirror *c.* is made to incline slightly over this hole, the reflection of the bubble is caught upon the silvered portion.

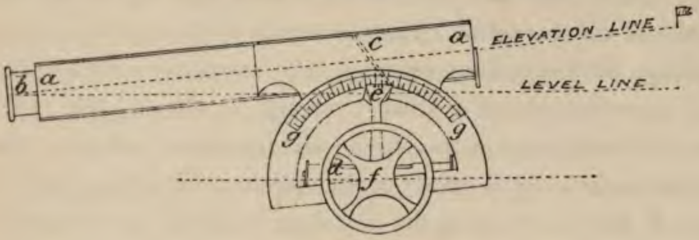


Fig. 6.

By placing the eye at *b.*, directing the telescope to an object, and by turning the wheel until the spirit bubble is seen reflected in coincidence therewith, the required angle will be found, as given by the reading of the vernier upon the arc.

The clinometer being like the optical square, a hand instrument, it should not be used where great accuracy is required; but it is admitted by all who have used it to be pre-eminently adapted to chainmen's work, and being simple in construction, as well as handy in the field, it is becoming a universal favourite.

In observing with this instrument, as also with the theodolite, it is necessary that the sight be taken parallel to the surface of the ground; that is to say, to a point situated at an equal distance from the ground as that at which the instrument is held. The simplest way to manage this in the field is, for the observer to take his sight to the brim of the person's hat, who may be stationed at the required point: provided of course that the two persons are of the same stature.

THE TAPE LINE.

The tape, or box line, is another very handy pocket instrument. It should be constructed of the best wire tape, 150 links in length, graduated in links upon both sides, and encased in a strong box, with the ring secured to the tape by leather fastenings. Most tapes are made so as to shew links upon one side, and feet on the other. This no doubt is very convenient, but through confusion, has sometimes been known to cause mistakes. As tape lines have a great tendency to shrink, they should never be used except in extreme circumstances, otherwise than in the measuring of offsets or dimensions. The tape should be tested frequently upon the standard, and the error carefully noted down, and allowed for in the measurements. The standard of a tape is to the outside of its ring. When in use in the field, it will be found most convenient to carry the box in the left coat pocket, and the line coiled in hanks in the left hand, inserting the little finger through the ring. In windy weather the line is very apt to snap, and should be held down close to the ground when extended out.

SIGHTING RODS.

Sighting rods, poles, or ranging rods as they are variously called, are most necessary articles in the field work of a survey. Their use is simply to cause the position of certain points upon the ground to be visible from a distance. They should be constructed of well seasoned timber, free from cast, about 5 ft. long, and 1 inch square, pointed at one end, and mounted with a piece of white calico so as to form a sort of flag.

In order to be able to identify certain points amongst a number of others so marked, it will be found a good plan to have some flags of red and white, or black and white calico to stick in at these points. Upon beginning a large survey, it is laways best to lay in a sufficient supply of these rods, say from

40 to 50, as it will be found that numbers get broken through accident or stormy weather.

A few rods of finer finish, such as those used by town surveyors or architects, may also be added. They should be of hardwood, ten links in length all over, planed to the octagonal, and the points protected by iron shods. Each link of their length should be painted alternately black and white, and pieces of tape sewn on to the flags for fixing to the rods. Rods of this description will be found very handy for sticking into hard ground, or ranging for intermediates. Three or four long stout poles are also required for the triangular stations, and large black-upon-white flags provided for each in order that they may be seen from a great distance.

Surveyors often err in not having a sufficient number of flags for the work, and many a check bearing which otherwise would have been observed, is discarded or forgotten altogether, simply because the small supply of rods would not admit of one or two being left behind for the indicating of these points. A considerable deal of time may be saved by setting out a large number of rods at first, instead of having the assistants continually away from the scene of operations, bringing forward poles which are no longer of use in certain positions, or having to proceed forward every few minutes to lay out fresh lines. To plant, *i.e.*, to stick up a sighting rod properly, it is necessary to place it perfectly perpendicular, otherwise the bearing, if taken to the top of the rod, will not be the true bearing of the bottom or point to be observed. In order, therefore, to have the rod perfectly perpendicular, a plumb-bob line should be applied to it upon all sides. If the string is suspended at arm's length, and held a little back from the pole, the true vertical position which the pole should occupy may be easily found. The rod is then hammered into the ground firmly, and the point marked by three stones placed round the bottom, in case the pole should accidentally be disturbed.

III. THE THEODOLITE: ITS ADJUSTMENTS, AND HOW TO USE IT.

While it is perfectly possible to execute a survey of comparatively small extent by the chain alone, it becomes quite impracticable to do so in the case of a large survey. The reason of this is, that the requisite tie or check lines for the verification of the results in a large survey would be of such a length that they would occupy a considerable time in measuring, and would be liable to as great error in themselves as the work they were meant to check. Again, in the case of town surveying, where it is impossible to obtain check lines, there would be no way of fixing or tying the work with assurance of accuracy; in short, the work, however accurately treated, would be of little or no value.

In order to illustrate the meaning of the two terms, check bearing and tie lines, which will be of frequent occurrence in this work, take for example the following:—

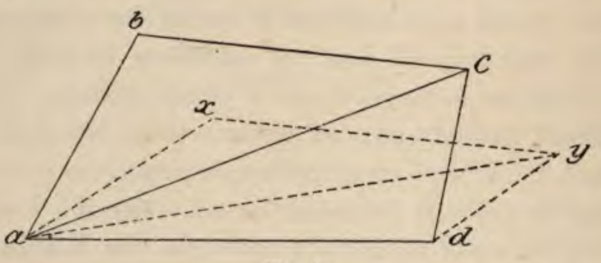


Fig. 7.

Fig. 7 represents a trapezium *a.b.c.d.*, the sides of which have each been carefully chained; it is however possible to have a figure as *a.x.y.d.*, composed of equal lines upon the same base *a.d.*, while at the same time occupying an entirely different position. But as the chief object of a surveyor is to ascertain and fix upon the plan the true position of these lines, it would become necessary for him to fix one point at least in this case. Let *c.* be the point proposed to be fixed. There are two methods of accomplishing this; the first is, to observe the bearing *A.C.*, and the second, to chain the line; but the best way is to do both. It is evident that, if the distance *a.c.* be measured, the figure cannot assume the position of the dotted lines, because the diagonal *a.y.* would be longer than *a.c.*, consequently, if the distance only had been measured, the line *a.c.* would become what is termed a tie line. Now, in the triangle *e.f.g.*, (Fig. 8), where the three sides have been measured, the case is slightly different, because, if these sides have been accurately chained, the position of the lines, with relation to each other, cannot be altered; but in order to check this assumed accuracy, if a tie line *f.h.* be measured, it follows that the areas of the triangles thus formed, viz., *e.h.f.*, and *f.h.g.* ought to be together equal to the whole area *e.f.g.*—thus shewing the practical value of the tie line *f.h.* It will be seen that, in Fig. 7, a similar check may be applied by measuring the other diagonal *b.d*, as the areas of the two triangles formed by this line should be together equal to the two already cut off by *a. c*, otherwise the chainage has not been uniform or consistent.

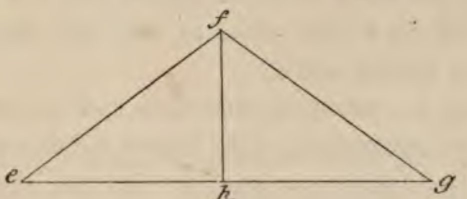


Fig. 8.

Now, the term check bearing means—an observation to or from a point (which has been partially fixed from another point), the position of which is known by some other means. Thus, observation of the lines, *a.c.* or *f.h.*, by means of a theodolite or angular instrument, would manifestly fix the true position of either of the figures; and would consequently be called a check bearing.

A surveyor should thus have constant resource to tie lines or check bearings; and, although the use of the theodolite renders unnecessary a large proportion of the internal tie lines (so indispensable in chain surveys), the fact should not be lost sight of that, where tie lines can be advantageously run, they ought to be taken advantage of; also remembering that a well chained tie line taken from the proper spot, and in a proper direction, will fix a point quite as well, and often far better than a check bearing would.

Traversing, *i.e.*, surveying with chain and theodolite, may be defined as the ascertaining of the length and direction at which one point is situated from another. In using the word direction, or more correctly speaking, *bearing*, a normal point is assumed. This in surveying, is the cardinal or north point, and all bearings are observed in relation to it. The theodolite is therefore an instrument for registering the angle which a survey line makes with the north.

The circumference of a circle is divided into 360 equal parts, called degrees, which are sub-divided into 60 equal parts, called minutes, and again into 60 equal parts, called seconds. Each quadrant of a circle is equal to 90 degrees; therefore, this standard of degrees, minutes, and seconds, furnishes a scale for the measurement of angles.

Let the lines *a.b.* and *a.c.* in Fig. 9, meet at the point *a.*, forming the angle *b.a.c.*—the magnitude of the angle is represented by the number of degrees, minutes, and seconds contained in the arc *d.e.;* and it is the province of the theodolite to furnish this result. In order, however, to obtain the bearing of these lines in relation to the north, assuming the point *N.* to

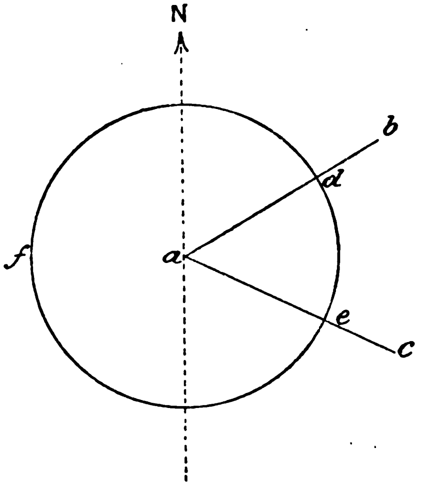


Fig. 9.

be north, the angle *N.a.b.* must be ascertained; this angle is then called the bearing of *a.b.* Similarly, the angle *N.a.c.* is termed the bearing of *a.c.* Surveyors term the North 360° or zero; and a glance at the following figure (Fig. 10), shews that East bears

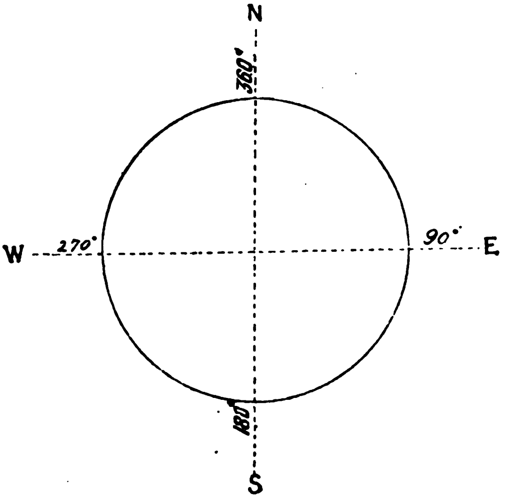


Fig. 10.

90°, South 180°, and West 270°. So that all observations with the theodolite have direct relation to the north or meridian line.

The theodolite consists of a metal plate, supported in the centre horizontally by a hollow cone nicely fitted into a socket, which admits of perfect freedom to revolve. The circumference of this plate is bevelled off, and faced with silver, upon which the angular graduations are engraved.

This plate is commonly known as the *lower plate.*

Through the centre of this hollow cone a solid cone passes, and is fitted with great exactness; it carries the centre of another plate, usually known as the *upper plate,* which is made to rest by its own weight upon the lower one, and is also free to revolve horizontally on its own axis; so that, while the two plates are held firm vertically by their cones, they are free to move in azimuth, in the same or opposite directions. Upon the edge of the upper plate a small silver surface is fixed, upon which a pointer is engraved, and, being in the same plane with the bevelled edge below, by turning the upper plate this pointer moves along the divisions of the lower plate, thereby giving the number of degrees or half degrees in the reading. Two upright standards are fixed upon the upper plate, which carry the trunnions of a telescope, and allow it to be moved in a vertical plane. Upon the top of this plate there are placed two small spirit levels set at right angles to each other, and attached to the telescope is another spirit level of larger size. Immediately under the telescope a vertical arc is fixed, which being attached to the same axis as the telescope, moves along with it. The edge of this arc is graduated into degrees similar to the lower plate, and is therefore capable of indicating angles of altitude or depression, assisted by a pointer fixed upon one of the upright standards of the telescope.

In order to make use of the lower plate in observation, it is evident that it must first of all be set in a certain position, while the other plate carrying the telescope and pointer moves round to indicate the angle. For this purpose a stopper or clamp is applied to the cone of the plate by means of a screw which holds it firmly in one position, while still allowing the upper plate freedom as before. Adjacent to this clamp a tangent or slow motion screw is fixed, acting upon the lower plate after it has been firmly clamped, so that it may be moved slowly and steadily into the exact position. As the upper plate requires to be fixed also, a clamp is provided for making it fast to the lower plate, accompanied by a tangent screw. A similar pair of screws are also provided for adjusting and fastening the arc and telescope. The whole instrument is terminated underneath by a ball and socket joint, acted upon by four screws made to work between the surfaces of two small metal plates, usually called *parallel plates;* and by turning these screws with reference to the spirit bubbles, the axis of the instrument may be made to assume a vertical position. A three-legged stand, upon which the instrument is screwed, completes a rough sketch of the theodolite.

In most theodolites used in this country a magnetic compass is fitted upon the centre of the upper plate, having the circumference graduated into degrees, which, being turned in relation to the lower plate, enables the surveyor to have a check upon his bearings. Owing to the deviations of the needle through local attractions, etc., the check really becomes of little value, but upon the whole the compass serves to add to the completeness of the instrument, and in some circumstances may prove of use. Within the apex of the legs a small hook is fixed, from which a plumb-bob can be suspended.

The instrument just described is what is termed a 5-inch plane theodolite, because the diameter of the lower plate is 5 inches, and the term plane is used to distinguish it from the "Everest" and "Transit" theodolites.

The telescope consists of a tube in which is fixed a combination of lenses forming an image of the object sighted within the tube, which image is magnified by a microscopical eye-piece.

Two eye-pieces are generally provided for each theodolite, the one shews the object inverted, and the right side as the left, the other shews it in its natural position; but in order to do so two more lenses are brought into use, which has the effect of lengthening the tube, thereby absorbing more light, to the detriment of the vision.

The use of the inverting eye-piece is consequently preferable; and although at first it is confusing to order a flagman to the right when he already appears too far in that direction upon the field of the telescope, the sense of inconvenience or awkwardness soon wears off in practice.

At the point of focus of the eye-piece and object-glass within the telescope, three cross wires are stretched, made to intersect at one point, and fixed upon a hollow disc, so that by looking through the telescope the intersection furnishes a fine point upon the field of view. As these wires may also be focussed upon directing the telescope towards any object, they can be made to coincide with any part thereof. Fig. 11. shews a section of the telescope with this *diaphragm*, as it is called, in its proper position. The cross wires usually consist of spiders' threads, as on account of the strong microscopic power brought to bear upon them, the finest material has to be used. They are fixed upon the frame with gum, and as already said, made to intersect each other at one point. The whole diaphragm may be moved laterally and vertically by means of the four capstan-headed screws *a.b.c.d.*, taking care that in tightening one, the opposite one has been previously slackened.

Some diaphragms are fitted with two hairs only, the one horizontally, the other vertically; but in observing a distant pole with this arrangement, the vertical hair is apt to hide the figure of the rod, so that, while the arrangement as in fig 11 leaves the point of intersection vertically exposed, the two angular wires are equally capable of shewing whether a pole is leaning out of its true upright position or not. Great care must be taken not to injure these hairs, either by carelessly inserting a finger into the tube, or exposing them to wind or rain.

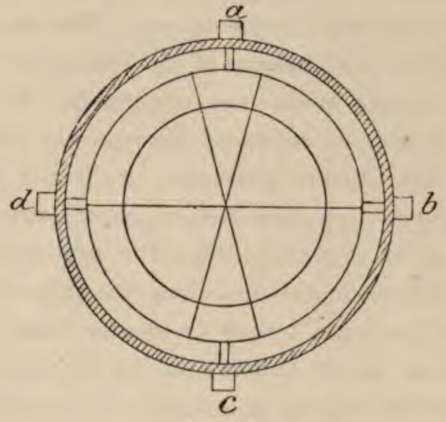


Fig. 11.

All the spirit levels of the theodolite are capable of being elevated or depressed by means of capstan-headed screws at each end. The telescope tube rests in what are termed Y's, within which it is kept by hinged clips closing over the tops of each, so that by undoing these clips the telescope may be turned either end for end, or about its longitudinal axis. The instrument should always be unscrewed from its legs, and placed within its case when it is required to carry it a long distance, because in carrying it over the shoulder there is a great strain upon the neck of the instrument, which has a tendency through time to bend the cone. As it is impossible to pack up the instrument at every observation, it is better when carrying it over the shoulder to keep it as nearly upright as possible. Upon no account should it be forgotten to place a ring upon the legs when carrying, as it prevents the large screws at the head of the legs from loosening; and all parts should be unclamped, and left free to move with the exception of the vertical arc.

Situated immediately alongside of the engraved pointer, and in conjunction therewith, is the *vernier*. This is a contrivance for indicating the exact balance of the reading in minutes and seconds, or in other words—the measurement of that part of the arc which may be contained between the pointer of the vernier and the primary graduation upon the lower plate; thus furnishing on inspection that which otherwise could only be guessed at or approximated. The arrangement consists in a second scale made to move along the side of the first, its divisions being a little longer or shorter than those of the primary. It is usually constructed by making a number of divisions upon the original scale equal to one more, or one less, than the vernier scale. The vernier of a theodolite is generally one part less than the same number of divisions upon the lower plate. In Fig. 12 the graduations of a vernier are shewn as magnified in their proper relations. As the two zeros are made to coincide, it will be seen that by looking along to the left, the thirtieth division upon the vernier coincides with the twenty-ninth upon the plate, so that $29 \times 30 - 30 = 29$ minutes, which substracted from half a degree (that is to say, each division of the lower plate), leaves one minute as the amount of closeness to which that vernier directly reads. In order to read a vernier, first look where its zero mark strikes the plate. If it coincides with a graduation precisely, the reading is exactly that of the division so indicated upon the lower plate. But if the zero of the vernier strikes between any two divisions, then note the next lower division of the plate for degrees, and look along to the left upon the vernier, until one of its graduations is found which is coincident with the graduation below; the reading of this upon the vernier is to be taken for minutes.

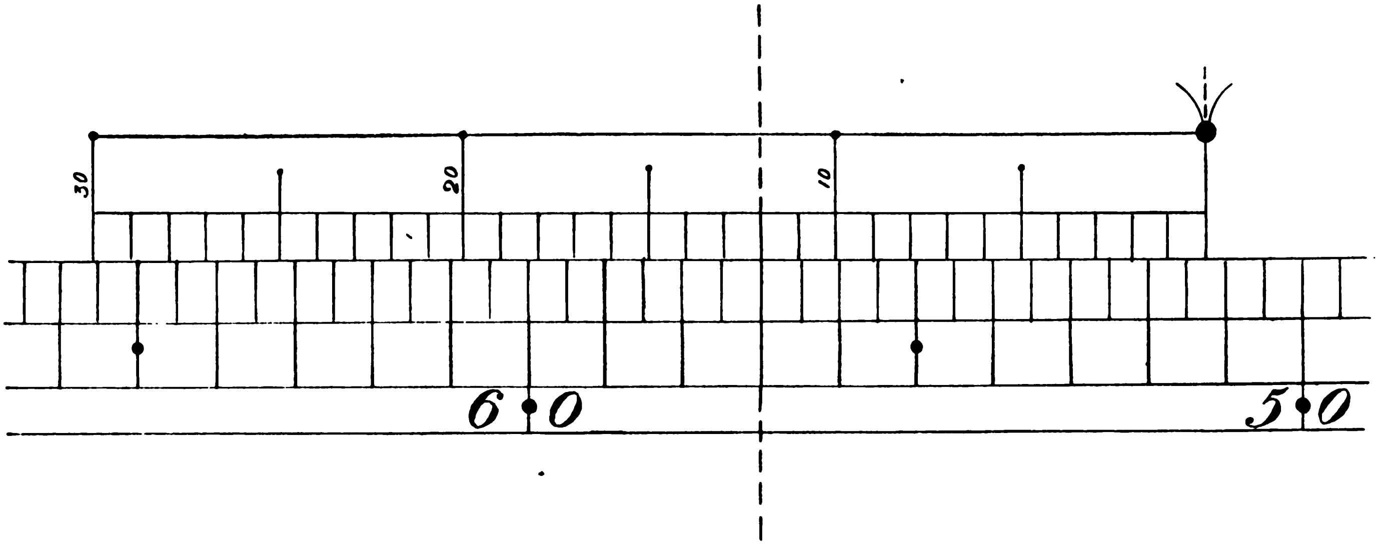


Fig. 12.

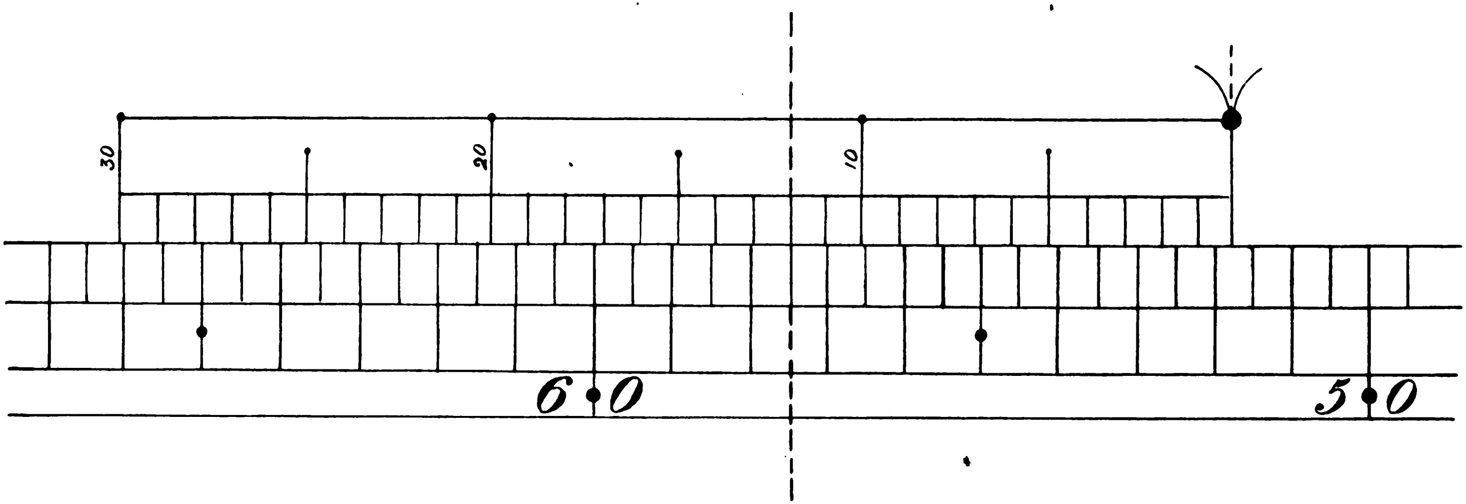


Fig. 13

In Figs. 12 and 13, it must be borne in mind that each degree is divided into two parts, that is to say, into thirty minutes each, and if the zero of the vernier should have passed one of these centre divisions without reaching the next degree marked, thirty minutes must be added to the reading of the vernier. In Fig. 12 the dotted line shews the point of coincidence to read 51.° 12′, but in Fig. 13 the reading is 51.° 42,′ which is the same reading upon the vernier, but with the half degree of the lower plate added thereto. Verniers may be constructed to read down to 20 and even 10″ by increasing the subdivisions upon the vernier and lower plate; but the requirements of an ordinary survey make such exactitude unnecessary, as it is perfectly possible to estimate the amount of extra seconds by the eye, and a smart surveyor will be able to read to within 15″ by means of the usual arrangement.

The following examples will shew how this may be effected. In Fig. 14 the reading of the lower plate gives 49.°, but in looking along the vernier for the balance it may be noted that either 7 or 8′ may be accepted, as both graduations appear in the same relation to the lower lines. This shews that the mean is the correct reading; so that the true observation is 49.° 07.′ 30″. It may be also noted that the differences of coincidence of the two lines immediately next these are the same, but if the observer can discern the one to the left to be the slightest shade smaller than the other, he may add 15 to the 30″ already obtained. But on the other hand, if the difference to the right is smaller, then 15 may be deducted from the original 30″.

In reading angles upon the vertical arc, extra caution must be used, as the altitude and depression graduations are similar, but situated upon different sides of the zero point; so that the readings are in opposite directions.

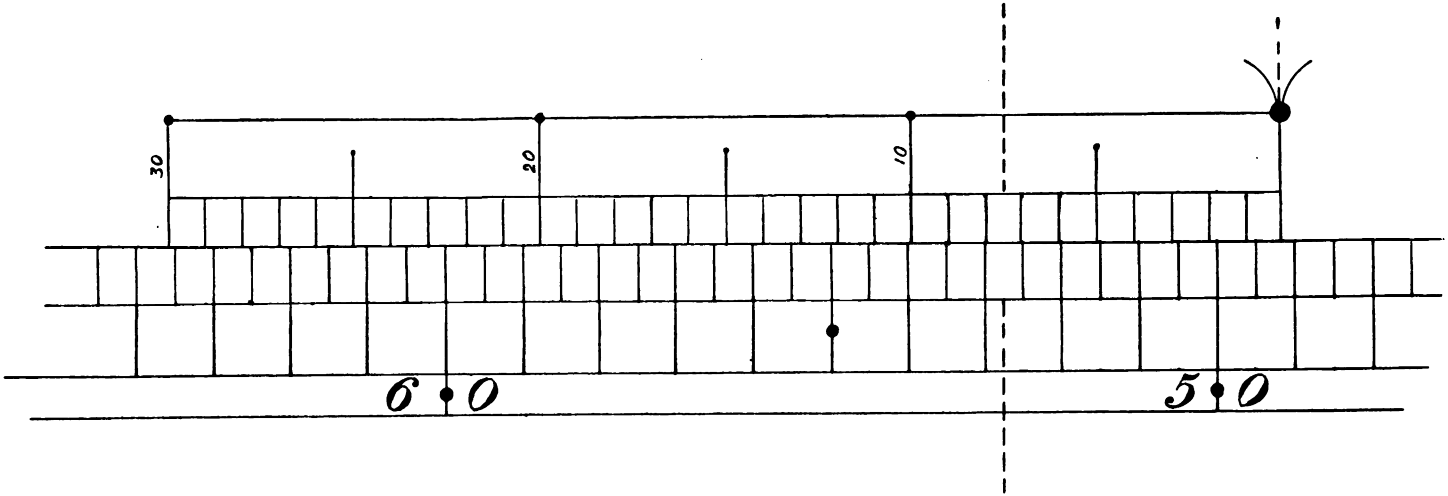


Fig. 14.

Fig. 15 is a true representation of a theodolite, and by means of the following description with reference to the letters upon the figure, the various parts may be recognised.

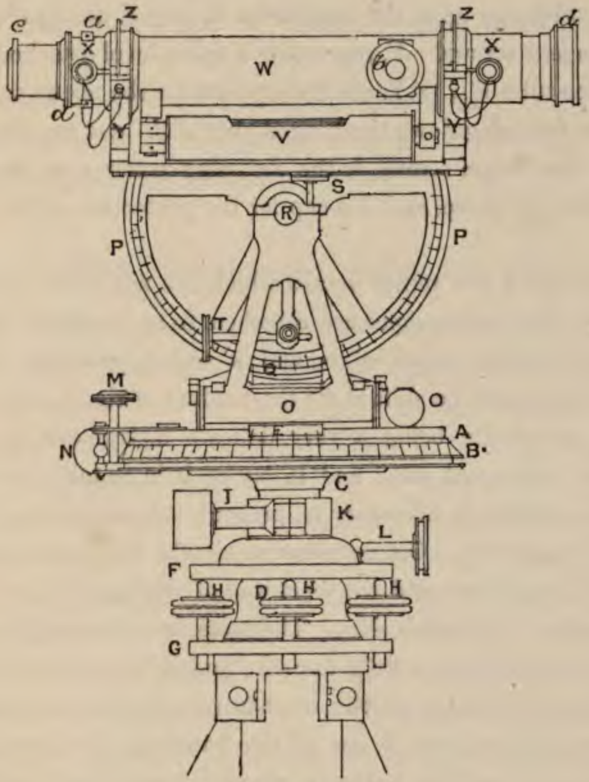


Fig. 15.

A. and *B.* represent the upper and lower plates, secured to the neck *C.*, and terminated in a ball working into the socket *D.*; *E.* represents the position of one of the verniers upon the upper plate, another vernier is placed at the other side directly opposite. *F.* and *G.* are the parallel plates between which the levelling screws *H.H.H.* work; *I.* is the screw for tightening the collar *K.* by seizing the neck *C.*; *L.* the slow motion or tangent screw of the lower plate; *M.* the screw for clamping the upper to the lower plate, and *N.* its tangent screw. *O.O.* are spirit levels placed upon the upper plate at right angles to each other; *P.P.* is the vertical arc, *Q.* its vernier, and *R.* the trunnion or bearing of the telescope upon the standards; *S.* and *T.* are its clamping and tangent screws. *V.* represents a spirit level attached to the telescope tube *W.*; *Y.Y.* are the supports for the telescope; *X.X.* pins for fastening down the hinged clips *Z.Z.* *a.a.* are the heads of the diaphragm screws, *b.* the focussing screw, *c.* an inverting moveable eye piece, and *d.* a cap for the protection of the object glass.

There are a few minor details which are not shewn upon the figure. Two microscopes are attached so as to range over the verniers of the upper plate, and a socket provided so that one of them may be shifted for the reading of the vertical arc.

The theodolite being a very delicate instrument, must be handled with great care; and in laying it in or taking it out of its box, caution is necessary to prevent it from getting jerked about in any way. On no account should the instrument be allowed to get wet, and the best way of drying it is to use a soft cloth, or a chamois skin. The motions occasionally require to be touched with a little fine oil; and if the surveyor has a sufficient knowledge of the mechanism of his instrument, he may be able to keep it out of the hands of the optician by taking some of the parts to pieces, cleaning and replacing them.

ADJUSTMENTS.

Every surveyor should not only understand the principles and mechanism of the theodolite, but should also be able to test its accuracy, and take the necessary means himself of orrecting any defects in adjustment.

Now, as this is considered one of the principal points upon which a surveyor should make himself conversant in the undertaking of large surveys abroad, or even at home, the subject is here treated in full detail.

These adjustments may be described under three distinct heads:—

1. The semi-permanent or parallax adjustment.
2. The permanent or collimation adjustment.
3. The temporary or vertical axis adjustment.

1.—*The Semi-Permanent or Parallax Adjustment.*

Before proceeding to deal with any angular result of the theodolite, the first thing to be done upon looking through the telescope is to observe if the diaphragm hairs are clearly defined to the eye without any reference to objects beyond, and if they are not so, by holding a white surface in front of the object glass, while at the same time moving the eye piece slowly out and in, the focus of the hairs will be obtained. Upon then directing the telescope towards an object, and bringing it into focus by using the milled headed screw on the side, it may be noticed that the cross hairs do not appear to identify themselves with the object, but flit or shift their position when the observer's head is moved, instead of remaining stationary upon the object —this is called the error of parallax. The remedy to be applied is simply again to move the eyepiece out or in, very slightly, until the flitting movement disappears. The effect of this latter operation is to make the foci of the eye piece and object glass exactly to intersect each other at the diaphragm hairs. As this adjustment has generally to be repeated every time the instrument is taken out of its case, it may be correctly termed a semi-permanent adjustment.

2.—*The Permanent or Line of Collimation Adjustment.*

The line of collimation means the line of vision which the eye takes in directing the point of intersection of the cross hairs towards a well defined object at a distance; and the adjustment of this line consists in causing it exactly to coincide with the optical axis of the telescope holders, and afterwards making this rectified line parallel to the true horizon. In order to effect this adjustment, set up the instrument, clamp the upper to the lower plate, and open the telescope clips. Direct the telescope towards a distant and well defined object by means of the hand only, bringing the cross hairs as near as possible to the object; then clamp the lower plate firmly with a steady turn of the lower clamp screw, and by means of the accompanying tangent screw, cause the hairs to coincide. Treat the vertical arc in a similar manner, finishing by clamping it firmly, making use of its tangent screw also. Having by these agencies got the centre of the cross hairs to coincide with the fine point in the distance, and also the various parts firmly clamped, proceed to turn the telescope half round upon its longitudinal axis, and then notice the result by looking through the telescope. If the points do not still remain coincident, half the deviation must be corrected by the tightening and slackening of alternate diaphragm screws, and the other half by turning the screw of the vertical arc. This operation requires to be repeated until the points are seen to coincide in both positions of the telescope. Now turn the telescope a quarter round; if the coincidence does not still exist, correct half the difference as before, by the diaphragm screws; but in this case the other half must be corrected by means of the lower plate tangent screw. Repeat both sets of operations if necessary, until by turning the telescope in all the four positions, no apparent difference is seen to exist at the point of intersection.

Having thus procured the line of collimation of the instrument, it has next to be made parallel to the true horizontal line, as indicated by spirit level. The method is as follows:—Unclamp the lower plate and the vertical arc, and with the hand, place the telescope as nearly level as possible, the clips still remaining open. Then clamp the vertical arc firmly, and bring the bubble of the telescope level to its centre by making use of the tangent screw. Gently lift the telescope out of the Y's, and replace it end for end. Notice now if the spirit bubble upon the telescope settles in the centre; if it does not, try the telescope in its original position again, so as to make sure that the vertical arc has not shifted. If the level registers the same as before, shift the telescope again into its second position, and proceed to correct half the error by means of the screw which fixes the level to the telescope, and the other half by turning the tangent screw of the vertical arc. Repeat these operations until the spirit level remains the same in both positions—the line of collimation has then been placed parallel to the true horizontal line. Now, it follows, that when the true horizontal line has been fixed, the vernier zero of the vertical arc should exactly coincide with the zero of the limb; but if it is found to be otherwise, the amount of difference is generally noted down, and termed the index error. It will be noticed that in the application of this error to the vertical angles, the difference must either be added to altitudes and deducted from depressions, or *vice versa* for every observation taken; but although it is correct to do so, it is somewhat difficult to manipulate. For example; the question as to whether the index error must be added or subtracted from an angle of depression, depends (1) upon which side of the limb the error lies; and (2) upon the position of the telescope in its Y's. Suppose the error to lie upon the side of the zero towards the eyepiece of the telescope, it would have to be subtracted from angles of depression and added to altitudes; consequently, if the telescope be placed end for end, with the same conditions, the application of the index error must necessarily be reversed. So that, if a change of this kind in the position of the telescope had nót been taken into account, an error of double the index error is made in the reading of the vertical angle from one point to another. A second and better way of getting over the difficulty arising from an index error, is to observe from both stations, so that as the one observation is altitude and the other depression—the average reading will be the true angle. The third, and simplest way of getting rid of the index error is, to slacken the screws which fasten the vernier to the instrument, and as these screws work through elongated holes, the vernier may be shifted and fixed with the aid of a microscope, into its true position; so that the instrument is made to have no index error, consequently no corrections are henceforth necessary.

The next permanent adjustment consists in making the circular plates parallel to the line of collimation:—Place the instrument as nearly level as possible without touching the screws; fasten down the clips of the telescope, and release all the clamp screws, with the exception of that of the lower plate. Then turn the upper plate round until the telescope points over a pair of parallel plate screws, as nearly level as possible; clamp the vertical arc, and level the telescope bubble accurately, by using the tangent screw; lastly, reverse the upper plate, turning it through 180°. If the bubble does not now return to the centre of the tube, correct half the difference by means of the vertical arc tangent screw, and the other half by tightening the parallel plate screws over which the telescope points. Repeat the operation in the opposite direction until the correct result is obtained, and then turn the screws attached to the levels upon the upper plate until the bubbles in each are also in their centres; turning the plate round, and adjusting them over each pair of screws: the instrument may now be considered in permanent adjustment.

3.—*The Temporary, or Vertical Axis Adjustment.*

Set up the instrument so that the lower parallel plate is as nearly level as possible to the eye, and unclamp all the parts; then proceed to manipulate the lower plate screws as shewn in Fig. 16.

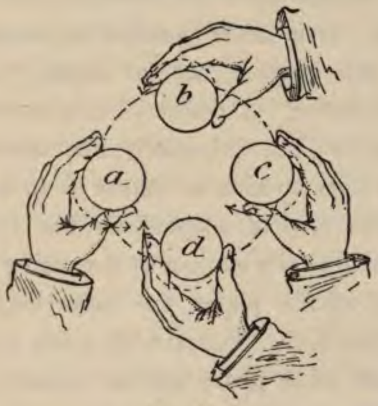


Fig. 16.

Having turned the instrument until one of the lower plate spirit levels rests immediately over the screw *d*, turn the two screws *a* and *c*, as shewn, simultaneously, but in opposite directions to each other, as indicated by the arrow heads, until the bubble lying over *d* is brought to its centre; then, as the other spirit level lies immediately over *a* or *c*, turn the remaining screws *b* and *d* in the same manner until this level also is adjusted. The last half of this operation will generally cause the first bubble to come again into error, so that both levellings must be repeated again. Then turn the instrument through 180°, and proceed to correct half the error; in shifting the plates back again to the original position, the desired effect should be attained, or nearly so; then turn the plate through

90°, and afterwards 180°, so that each level has been adjusted in all positions. The theodolite may now be considered in perfect adjustment, and capable of registering angles of azimuth and altitude from this spot with perfect precision. The adjustment last described requires to be made every time the theodolite is planted, but by a little practice the surveyor will be able to apply it quickly and easily.

A glance at a theodolite will shew that a good deal of its accuracy must depend upon the precision with which its arcs are constructed. It is never possible to attain mathematical exactitude in this respect, but, of course, the nearer it is approached the better. A defect of this nature is called the error of graduation; but fortunately, like other errors, it may be overcome, at least to such an extent as to cause no appreciable errors. The method adopted for the correction of this error being similar to the manner of correcting another defect, viz., eccentricity of the plates, the same description will be made to stand for both. Eccentricity exists when the centres of the upper and lower plates are not coincident. Take Fig. 17 below as an illustration.

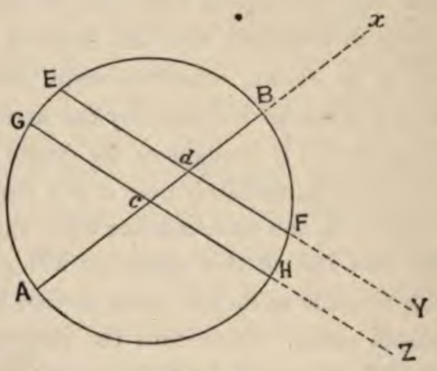


Fig. 17.

Let *C.* be the centre of the lower plate, and *d.* that of the upper;

and let *A.c.B.x.* represent a bearing, and *E.d.F.Y.* another bearing:—It is required to obtain the true angle contained by these two lines. Make the line *G.H.* parallel to *E.F.* passing through the centre *c.* Then the angle *A.c.G.* will be the angle contained by the arc *A.G.* or *H.B.* Now the eccentricity of the plates has caused *A.E.* and *B.F.* to be read, which were unequal; but as *G.H.* is parallel to *E.F.*, the arcs *G.E.* and *H.F.* are also equal. Therefore *A.E.*—*E.G.* $=$ *B.F.* $+$ *F.H.*, that is to say, the arcs *A.G.* and *B.H.* are equal to one another, so that if the half sum of the two arcs *A.E.* and *B.F.* be taken, it also will equal *A.G.* or *B.H.* But to apply this rule it is necessary to have at least two verniers, and in order to provide against the eccentricity error, both of these verniers should be used in the reading of an angle or bearing. The graduation error is got over in the following way. After having taken a series of observations from a certain point, unclamp both plates, and dividing the circle into three parts, add one of these parts, *i.e.* 120°, to the first angle read; set the plates to the new bearing, and clamping the instrument in its first position, again observe all the points, taking the readings of both verniers for each. Repeat this operation once, so that there are six readings for each sight; the mean reading of each may be considered alike free from error of graduation and eccentricity. Some surveyors divide the circle into four parts of 90.° each, giving eight readings for each bearing; but for ordinary work three divisions, that is to say, six readings, are considered sufficient. In traversing, it is generally unnecessary to read both verniers at every point, especially if the instrument is a good one, because the real accuracy of a traverse lies in the hands of the chainmen.

How to use the Theodolite.

Suppose six ranging rods to be set up at various points and carefully placed in positions over which the instrument may be set and plumbed to the perpendicular, it is required to survey this traverse with the theodolite, taking for granted that it is possible to see from point to point.

Proceed to point II., Fig. 18, and having lifted the rod so as not to disturb the station hole, lay it aside out of the way, and set up the instrument over the hole by spreading the legs out so as to cause their feet to be situated somewhat in the form of an equilateral triangle upon the ground. In order to save time in levelling and secure a firmer foundation for the axis of the instrument, move the legs until the lower parallel plate is in a horizontal position. Suspend a plumb-bob from the hook underneath, and again move the legs until the plumb-bob rests in the hole, or immediately over it. Then with a steady motion press each leg down to a firm foundation in the ground, being careful not to shove them in from the opposite side of the instrument in a jerky manner, as this is certain to damage the instrument. Having got the plumb-bob immediately above the hole, unclamp the vertical arc, and adjust for parallax; then follow with the temporary adjustments upon the completion of which the instrument is ready for use.

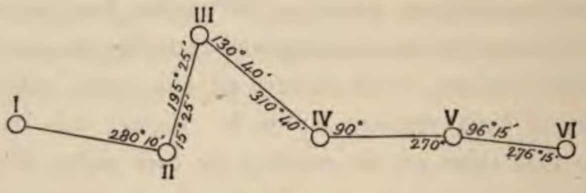


Fig. 18.

Now it may be assumed, that the bearing II.-I. is given; that is to say, its bearing with relation to the true north. Then proceed by clamping the upper and lower plates together so as to cause the vernier to read the true bearing, using the tangent screw. Then catch hold of the rim of the lower plate, and turn the instrument gently round until the telescope points towards the pole at I. Look through the telescope, and clamp the lower plate, causing the centre of the cross hairs to cut the foot of the pole by using the lower tangent screw. Again read and compare the bearing to see that it has not altered, and releasing the upper plate clamp, catch hold of the instrument by the telescope standards, and turn it gently and steadily round to the right towards point III., taking care not to "overshoot the mark." Apply the eye to the telescope, adjust the focus, and bring the hairs to coincide as nearly as possible by the hand, being again careful not to overshoot the mark. Then clamp the upper plate, and using its tangent screw until its intersections coincide, read off the bearing upon the same vernier which was used in setting. After this unclamp the upper plate, and turning the instrument round to the right, again set and clamp upon pole I. Read off the bearing (which should be exactly the same as the original), then unclamp and turn the upper plate again towards III.; read and compare the bearing with the former one, and if found to agree, the operation at point II. is complete. Now suppose that in reading back upon station I., the bearing differed 1 or 2′ from the original, it becomes necessary to set the vernier afresh by again using the upper tangent screw, and adopting the same course once more by bringing the telescope round upon III.; this reading will in all probability be different from the last by exactly the same amount as the former error, so that it will likely prove correct. But whether this is the case or not, complete all the observations, and finish off at the same place as before. If the bearing to III. should always read the same, and yet the closing one agree with the original, it is an indication that the lower clamp screw has been allowing the cone to slip, and in order to remedy this, the best plan is to unclamp the lower plate altogether, and do the work afresh from the beginning. The frequent cause of this error is, that the screws which fasten the instrument to the top of the legs have become slack, and admit of a slight motion—the obvious remedy is to tighten these screws.

Proceeding to station III., set up and adjust the instrument, and referring to the field-book for the bearing from station II., add 180.° to it; setting the plates to this bearing, proceed in exactly the same manner as at station II. Referring to Fig. 18, it will be seen that the bearing from station II.-III. was 15.° 25.′, but from point III., II. bears in exactly the opposite direction; therefore, instead of 15.° 25.′ being the bearing from III.-II., 15.° 25.′+180.°=195.° 25.′ is the bearing of II. from III., so that the upper plate must be set to this bearing. The following rule arises from the above:—If the original bearing is less than 180.°, add 180.° to it, but if more, subtract 180.° from it for the back setting. A glance at the opposite vernier will check this calculation. The vernier generally used when traversing is the one placed upon the left side of the instrument, looking in the line of the telescope, and is called vernier *A*; the other one is called vernier *B*, but either of the two may be used, provided it is adhered to throughout. The same operations are repeated at every point, thus always setting the plates upon the converse of the forward reading from the last point for the back reading, clamping this bearing upon the back rod, and turning the instrument to the right to the next forward station, then checking back again upon the back pole.

It is evident that in order to cause all the bearings to have direct reference to the meridian, the lower plate must be made fast at each point in this true position, so that the upper plate by turning upon it, measures each angle therefrom. A glance at Fig. 18 will shew that the same result can be found by observing each angle separately. For example: at station II. clamp the plates upon zero, and turning round upon III. the interior angle will register 95.° 15.′ Proceed to III., again set 360.° as before upon III., and read the interior angle 64.° 45.′ to IV., and so on till V. is reached. Then it will be found, that by assuming the bearing III.-II. to be the same as before, viz., 280° 10′, the true bearing of any of the lines may be ascertained by adding thereto the interior angles when a line bears to the left of the traverse, and deducting the same when the bearing is to the right. This is a complicated procedure, and will not stand comparison with the other method, which actually causes the instrument to compute these results itself as it goes along. Private surveys in this country are generally constructed in relation to the magnetic north. One reason for this is, that if the vernier is set in relation to the compass upon the theodolite at starting, the surveyor has a check upon his observations; because, provided there are no local attractions of the needle, it should read the same bearing as shown upon the limb of the instrument at each point. Another reason is, that by knowing the exact magnetic variation, the true north may be laid down upon the plan. To place the compass in relation to the limb of the instrument, after adjustment set the plates to 360.°, clamping them together; then turn the instrument round until the north end of the needle points to zero upon the compass graduations; clamp the lower plate, and if necessary finish by using the lower tangent screw. Then unclamp the upper plate and proceed with the survey. Provided no wire fences, iron rails, or other objects having attraction for the needle are near, this ought to form a check for the bearings, to within half a degree.

IV. THE BASE LINE: ITS MEASUREMENT, AND RELATIVE TRIANGULATION.

As the base line is the foundation from which the triangulation of a survey is calculated, upon it the accuracy of all future work rests, so that unless its length may be thoroughly relied upon, the surveyor can have no security that his other work is correct. Further, in the event of an error arising in its measurement, the blunder would be carried from triangle to triangle in a ratio proportionate to the size and conditions of the triangles, and their distances from the base, until outlying triangles would become unfit for the purposes for which they were intended, both with respect to distance and bearing. Many methods have been tried by which to attain perfection in this matter, but a perusal of the work called "A Trigonometrical Survey of England and Wales," will alike shew the great importance and perfection necessary in the chainages of base lines.

For any ordinary survey, the steel band, treated with proper care, will be found perfectly suitable, provided it has been adjusted upon a reliable standard before and after use. In the plan given at the end of this work will be found a complete survey upon trigonometrical principles, and to which all the following examples have reference.

Let the points *A.* and *B.* represent the selected extremities of a base line. Cause the chainmen to measure from *A.* to *B.* with the tested chain, upon the principles set forth in Art. I., noting down the length, as read to one decimal of a link. Then chain back to *A.* from *B.*, and again from *A.* to *B.*, noting down the three results. Their mean will be the true distance thus:—

A. to *B.* =	2376·4	1st	measurement
B. to *A.* =	2377·6	2nd	ditto
A. to *B.* =	2376·7	3rd	ditto
3 |	7130·7		
Mean distance	2376·9		

If, upon testing the chain after the 3rd measurement it is found to have stretched, then ascertain exactly the amount of difference, which, divided by the number of chains in the base—upon the assumption that the error has been progressive—will represent the amount to be added to each chain length for the correct distance. Before chaining the base, the theodolite should be set up at the point *A.* in order to range poles for the purpose of enabling the chainmen to keep the straight line. To do this, send forward an assistant with poles in the desired direction, clamp the lower and upper plates of the instrument so that the cross hairs intersect upon *B.*, then the assistant, by means of signals from the observer, may be put into general line, and by making use of the telescope vertically, the rod can be finally set with precision.

TO FIND THE TRUE MERIDIAN.

At the commencement of the survey set up the theodolite over one of the proposed trig points about one hour and a half before noon. Carefully adjust the instrument, and, clamping the plates without reference to bearing, cause the horizontal hair in the telescope to coincide with either the upper or lower limb of the sun, using a coloured glass for the protection of the eye. Clamp the lower plate and vertical arc, and by means of the tangent screws attached to each, apply the cross hairs to the edge of the sun. Assuming that an inverting eye piece is used, Fig.

19 shews the position the sun would occupy within the telescope, supposing that the lower and left limbs were observed.

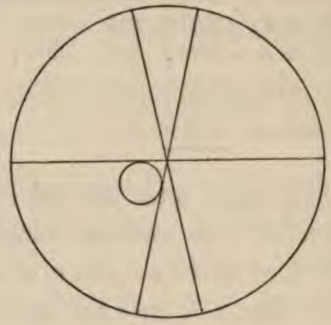


Fig. 19.

Note down the reading of the horizontal and vertical arcs; a few minutes afterwards release the upper plate and vertical arc, keeping the lower plate still clamped, and note down both vernier readings as before; a little later take another set of readings, and finish the observing in ascent by releasing the upper plate, keeping the vertical arc clamped in its last position. Now, after twelve o'clock, when the sun is in its descent, move the upper plate round, following the sun, and on looking through the telescope clamp the upper plate when it appears nearly to coincide as before, but with this difference, that—while adhering to the lower limb throughout, the right limb must be substituted in descent; or, if the right was observed in ascent, the left must be taken in descent, otherwise an angular error, equal to the sun's semidiameter, would be committed. Use the upper plate tangent screw to cause the hairs to follow until the exact coincidence takes place, then note the reading of the horizontal arc. Release the upper plate, and, setting the vertical arc to the second last elevation noted, again follow the sun horizontally until it arrives in a similar position with respect to the cross hairs, and carefully note the horizontal reading. Thus observe the horizontal reading at each similar altitude, then the True South will be the point upon the horizontal arc as given by the mean horizontal readings, or at least as nearly so as necessary. Of course if the observer is in South Latitude the mean will be the True North.

By observing a circumpolar star, in a similar way, the true meridian may be found. In order to see the hairs in the telescope at night, it is necessary to cast a faint light into the object-glass by reflection from a white surface. Although observations by night do not admit of the true bearing of a survey line being fixed with the same ease with which it may be done in daylight, the star gives the result required without any necessary correction, whereas the sun, as observed, owing to the obliquity of the eliptic has a slight variation, entailing a complicated correction for certain seasons of the year. But the first method explained is found sufficiently accurate for surveys of ordinary dimensions.

Thus, the instrument being set up at *A.*, and the following observations taken in the manner already set forth (I. II. III.), representing each observation in ascent and descent; the method of finding the required bearing is thus—

	ASCENT.	ALT.	DESCENT.	AZIMUTHS.	HALF AZIMUTHS.
I.	19° 30′ 00″	(11° 16′)	136° 30′ 00″=	117° 00′ 00″=	58° 30′ 00″
II.	30° 15′ 00″	(13° 24′)	126° 02′ 00″=	95° 47′ 00″=	47° 53′ 30″
III.	46° 30′ 00″	(14° 10′)	109° 50′ 00″=	63° 20′ 00″=	31° 40′ 00″

	DESCENT.	AZIMUTHS.	
Therefore I.	136° 30′ 00″ –	58° 30′ 00″=	78° 00′ 00″
II.	126° 02′ 00″ –	47° 53′ 30″=	78° 08′ 30″
III.	109° 50′ 00″ –	31° 40′ 00″=	78° 10′ 00″
		Mean bearing of Meridian=	78 06′ 10″

Now, upon observing the point *C.*, it is found to read 272° 32 55″, so that the contained angle between it and the meridian is 272° 32′ 55″ – 78° 06′ 10″ = 194° 26′ 45″. But as the meridian adopted was South, 180° subtracted from it gives 14° 06′ 25″—

the standard bearing of the survey with relation to the true North.

During the operation of finding the meridian, the chainmen should be despatched to set up long rods at the proposed "trig" points, so that no time may be lost. As each of these points have their positions fixed from the base, they serve as test points to which the traverses should close, so that the surveyor from time to time is enabled, by connecting to these points, to make the necessary corrections. What is known as triangulation is the manner adopted in fixing them. This consists in causing the lines between these points to form the sides of triangles, one of which has the base for a side, from which all the others are computed by the rules of trigonometry. In order to obtain good results, all the triangles should be well-conditioned, that is to say, should be as nearly equilateral as possible; but as this is not always attainable, a limit may be drawn from the following rule, viz., no angle should be greater than 120°, or less than 30°. The effect of an ill-conditioned triangle is to increase the error in proportion to the difference of lengths of the sides. In surveys of large magnitude, it would be manifestly inconvenient to chain a base line of length equal to the sides of the proposed triangulation, because, as these sides range from 2 to 5 miles, a flat piece of country of this extent is rarely to be met with. The method generally adopted is to enlarge the triangles from the base somewhat after the manner shewn upon the Plan. "Trig" stations must be situated so as to be clearly visible from all their relative points, and for this reason it is best to place them upon eminences, so that from one point the others connected therewith may be seen sharply defined against the sky. In a survey such as shown upon the plan, they may also be made to occupy positions near to or upon any of the proposed traverse lines. It is well to consider all this before starting to lay off these points, as the surveyor, by making a general inspection of the whole ground beforehand may save himself a deal of time and labour. Preliminary inspections of this sort may be aided by the surveyor carrying a pocket sextant or prismatic compass along with him, so that by ascertaining the proposed angles of the triangulation he may satisfy himself as to whether they are suitable before taking further steps.

The assistants having planted the "trig" poles in a truly vertical position, must also make sure of their stability, because the poles must remain in that fashion throughout the duration of the survey. Referring to the plan, the "trig" points are indicated by the letters *A.B.C.D.E.F.G.* and *H.*: the base *A.B.* is drawn in red, and *A.* is the station at which the meridian was obtained, and from which the standard bearing *A.C.* was observed.

In extensive surveys, where triangulation has to be carried a great distance from the original base, another base is measured, and being applied to the side of a triangle far distant from the base, checks the accumulated error. A base of this nature is called a *verification base.* It is evident that in the measurement of a Verification Base, its conditions must be the same as those of the Original Base before a comparison of results can be made: thus, for example, in Fig. 20. Let the arc $a.b. = b.c.$

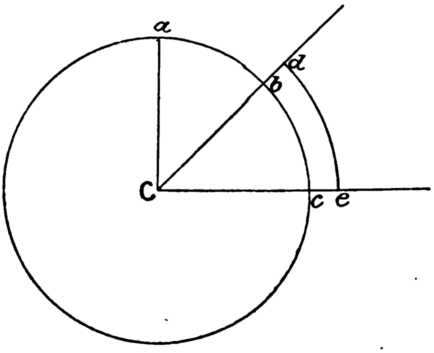


Fig. 20.

upon the surface of the earth at sea level, and let a base line be represented by the arc *a.b.*; then *d.e.* would represent the actual chainage of *b.c.* as elevated above sea level, and would be greater than *a.c.* or *a.b.* in proportion to its greater elevation. In a very extensive survey, therefore, where the difference of elevation between the base and verification base is considerable, a correction has to be applied; but in the present case it is sufficient to note this fact. The reason that a similar correction is not required for the angles in triangulation is, because the theodolite gives its angles in azimuth.

V. HOW TO OBSERVE AND TABULATE TRIANGULATION ANGLES.

Setting up the instrument at "Trig" *A.* in proper adjustment, proceed to rule columns in the field book for the observations, so that each point observed from *A.* has a column to itself headed by its letter. Referring to the following example of observed bearings in next page, proceed to write over the top of these columns "From *A.* set on *C.*, standard bearing 14° 26′ 45″;" clamp vernier *A.* to that bearing, and bring the hairs to coincide with the bottom of pole *C.*, clamping also the vertical arc and using lower plate and vertical tangent screws. Having accomplished this, read vernier *A.* once more, to make sure that it reads exactly; if otherwise, adjust, first by using the upper plate tangent screw, and then (upon looking through the telescope) by the lower tangent screw. Note down 14° 26′ 45″, the reading of vernier *A.*, in column *C.*, then read the opposite vernier *B.*, taking account only of its minutes and seconds, which is given as 26′ 30″, and place it also in the same column, immediately under the reading of vernier *A.* Next, unclamp the upper plate and vertical arc, and turn the instrument round towards *B.*, taking care not to overshoot the pole. Clamp the vertical arc, then the upper plate, and by using the tangent screws of each, cause the hairs to intersect the foot of pole *B.* Note down the reading of vernier *A.* = 69° 34′ 15″ in column *B.* as shewn, looking once more through the telescope to make sure of the coincidence; then read and note down vernier *B.* 33′ 45.″ Proceed in the same routine with the observations to *D.* and *F.*, being careful to turn the instrument always to the right, and not too far, because in bringing it back again there is the risk of shifting the lower plate through reversing the strain upon the clamps and cone—in fact, if the mark should be overshot, it is better to turn the upper plate completely round, somewhat on the same principle as in shifting the hands of a clock forward ten hours when it is already two hours fast. Should the instrument shew any signs of shifting after adjustment, the whole operation must be gone over again, both with respect to adjustment and observations thereafter taken from that particular point. Upon reading the last bearing from this point, viz., the *B.* vernier upon *F.* 2′ 45″, and noting in the column, undo all the clamping screws of the instrument, and, if necessary, make the temporary adjustments afresh; add ⅓ of the circle = 120° to the original standard bearing 14° 26′ 45″ = 134° 26′ 45″, and setting vernier *A.* to this reading, clamp the instrument upon *C.*, and proceed exactly as before, observing all the points as taken from this new standard, booking them in their proper columns. In this series, however, observe the angles of elevation or depression at each point, reading them to the foot of each pole, and noting them in the side column as indicated, being careful to state whether the angle is elevation or depression.

TABLE No I.

Observed Bearings from A set on C, Standard Bearing 14° 26′ 45″.

C	B	D	F	VERTICAL ANGLES.
° ′ ″	° ′ ″	° ′ ″	° ′ ″	
14 26 45	69 34 15	130 24 15	313 03 00	
... 26 30	... 33 45	... 23 45	... 02 45	
134 26 45	189 34 00	250 24 15	73 02 45	(Index error + 1′ alt.)
... 27 00	... 34 30	... 24 00	... 02 30	B.0° 05′ dep. - 1′ = 0° 04′ dep.
254 26 45	309 34 15	10 24 00	193 02 30	D.2° 20′ dep. - 1′ = 2° 19′ dep
... 27 15	... 34 30	... 23 45	... 02 45	F.1° 30′ alt. + 1′ = 1° 31′ alt.
14 26 50	69 34 12	130 24 00	313 02 42	
— 05	— 05	— 05	— 05	
14 26 45	69 34 07	130 23 55	313 02 37	

Observed Bearings from B set on A, Standard Bearing 249° 34′ 07″.

A	C	D	VERTICAL ANGLES.
° ′ ″	° ′ ″	° ′ ″	
249 34 00	310 13 00	193 44 00	
... 34 15	... 12 15	... 44 30	
9 34 00	70 12 00	313 44 15	A.0° 04′ alt. + 1′ = 0° 05′ alt.
... 33 30	... 12 30	... 43 45	D.2° 06′ dep. - 1′ = 2° 05″ dep.
129 34 00	190 12 00	73 43 30	
... 34 00	... 13 00	... 44 00	
249 33 57	310 12 17	193 44 00	
+ 10	+ 10	+ 10	
249 34 07	310 12 27	193 44 10	

TABLE No. I.—*Continued.*

Observed Bearings from C set on A, Standard Bearing 194° 26′ 45″.

A	E	F	H	B	D	VERTICAL ANGLES.
° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	
194 26 45	216 07 15	264 11 30	109 38 45	130 12 15	163 12 45	
... 26 ...	... 06 30	... 11 00	... 38 00	... 11 45	... 11 15	
314 26 45	336 06 45	24 11 45	229 38 30	250 12 00	283 11 30	
... 26 30	... 06 45	... 11 30	... 38 30	... 12 00	... 11 30	D 3° 50′ dep. - 1′=3° 49′ dep.
74 26 45	96 07 45	144 12 00	349 38 45	10 12 15	43 12 00	
... 27 15	... 08 00	... 12 00	... 38 30	... 12 30	... 13 00	
194 26 40	216 07 10	264 11 37	109 38 30	130 12 07	163 12 00	
+ 05	+ 05	+ 05	+ 05	+ 05	+ 05	
194 26 45	216 07 15	264 11 42	109 38 35	130 12 12	163 12 05	

TABLE No. I.—*Continued.*

Observed Bearings from D set on A, Standard Bearing 310° 23′ 55″.

A	C	G	B	H	E	VERTICAL ANGLES.
° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	° ′ ″	
310 24 00	343 12 45	5 23 00	13 44 00	61 01 00	259 57 30	
... 24 30	... 13 00	... 23 00	... 44 15	... 01 30	... 58 00	
70 23 30	103 12 30	125 23 45	133 44 15	181 01 30	19 57 00	A.2° 12′ alt. + 1′ = 2° 13′ alt.
... 24 00	... 12 30	... 23 30	... 43 45	... 02 00	... 57 30	C.3° 59′ alt. + 1′ = 4° 00′ alt.
190 24 15	223 13 00	245 24 00	253 44 00	301 02 15	139 57 30	G.4° 03′ alt. + 1′ = 4° 04′ alt.
... 23 45	... 12 45	... 23 30	... 43 00	... 01 30	... 57 15	H.1° 56′ alt. + 1′ = 1° 57′ alt.
310 24 00	343 12 45	5 23 27	13 43 52	61 01 37	259 57 27	E.0° 14′ alt. + 1′ = 0° 15′ alt.
− 05	− 05	− 05	− 05	− 05	− 05	B.2° 00′ alt. + 1′ = 2° 01′ alt.
310 23 55	343 12 40	5 23 22	13 43 47	61 01 32	259 57 22	

TABLE No. I.—*Continued.*

Observed Bearings from E set on C, Standard Bearing 36° 07′ 15″.

C	D	VERTICAL ANGLES.
° ′ ″	° ′ ″	
36 07 15	79 57 15	
... 07 30	... 58 00	
156 07 15	199 57 00	
... 06 45	... 56 30	D. 0° 18′ dep. - 1′ = 0° 17′ dep.
276 07 15	319 57 30	
... 07 00	... 57 30	
36 07 10	79 57 17	
+ 05	+ 05	
36 07 15	79 57 22	

Observed Bearings from F set on A, Standard Bearing 133° 02′ 37″.

A	C	VERTICAL ANGLES.
° ′ ″	° ′ ″	
133 02 30	84 11 30	
... 02 00	... 11 15	
253 02 45	204 11 30	
... 03 00	... 12 00	
13 02 30	324 11 00	A. 1° 46′ dep. - 1′ = 1° 45′ dep.
... 02 15	... 11 00	
133 02 30	84 11 22	
+ 07	+ 07	
133 02 37	84 11 29	

TABLE No. I.—*Continued.*

Observed Bearings from G set on D, Standard Bearing 185° 23′ 22″.

D	H	VERTICAL ANGLES.
° ′ ″	° ′ ″	
185 23 30	133 18 30	D.4° 20′ dep. - 1′=4° 19′ dep.
... 24 00	... 18 45	
305 23 15	253 18 15	
... 23 15	... 18 00	
65 23 15	13 18 30	
... 22 00	... 18 15	
185 23 12	133 18 22	
+ 10	+ 10	
185 23 22	133 18 32	

Observed Bearings from H set on C, Standard Bearings 289° 38′ 35°.

C	G	D	VERTICAL ANGLES.
° ′ ″	° ′ ″	° ′ ″	
289 38 30	313 18 00	241 01 15	D.1° 37′ dep. - 1′=1° 36′ dep.
... 38 00	... 18 00	... 01 00	
49 38 45	73 18 45	1 01 30	
... 38 45	... 19 00	... 01 30	
169 38 30	193 18 45	121 00 15	
... 38 15	... 18 30	... 00 00	
289 38 27	313 18 30	241 0 55	
+ 08	+ 08	+ 08	
289 38 35	313 18 38	241 01 03	

Before accepting these results it is necessary to rid them of any existing index error, by applying it plus if the vertical angle is elevation, and minus if depression, or *vice versa* (see Art. III. index error). This is the method adopted in the Plan as shewn in the preceding Table, and also as extended in the separate table of altitudes below.

TABLE No. II.

Altitudes.

TRIGS.	CORRECTED MEAN RECIPROCAL ANG.	DISTANCE IN LINKS.	DIFFERENCE FROM EACH POINT.		TOTAL FROM SEA LEVEL.
			ALT. FEET.	DEP. FEET.	FEET.
D to	° ′ ″				250
E	0 16 00	4397	13·5	...	263·5
A	2 16 00	2201	57·5	...	307·5
C	3 54 30	3817	172·1	...	422·1
G	4 11 30	4941	239·0	...	489·0
B	2 03 00	2322	54·8	...	304·8
H	1 46 30	4092	83·7	...	333·7
A to					307·5
F	1 38 00	2866	53·9	...	361·4

Where it is not intended to observe reciprocals, in order to get rid of the Index error it is necessary to take a second reading. This is done by turning the horizontal plates through an arc of 180,° lifting the telescope out of the *Ys*. and laying it end for end. Should there be any doubt as to the exactitude of the line of collimation, take a reading in each position, with the telescope turned half round about its longitudinal axis. Generally speaking, one set of vertical angles is sufficient from each point where the reciprocals are also taken from the other extremities of the lines, so that it may be considered advisable to take them in the middle of the series, as shewn in the example, because the adjustments may then be considered more reliable. The index error upon the Plan is $+1'.00''$ in alt.

Upon completing the series, again unclamp, adjust, and add 120° to $134°\ 26'\ 45''=254°\ 26'\ 45''$, and proceed in the same way as before, finishing off at *F.* with the reading of vernier *B.* It may be here remarked that the vernier readings should be inspected two or three times after the bearing is booked.

Now find the mean readings which each of these bearings give: thus—$C.=14°\ 26'\ 50,''$ $B.=69°\ 34'\ 12''$ etc. By this it will be seen that there is an increase of $5''$ in the observed bearings of *C.*, so that this has not only to be deducted from bearing *C.*, but also from each bearing in the series, accepting each quantity so corrected as the true standard bearing from this point. The operations at *A.* being completed, and the standards fixed by calculation upon the ground, take the instrument forward to *B.* By referring to column *B.* it will be found that its standard bearing is $69°\ 34'\ 07''$ from *A.*, so that *A.* must bear the converse from *B.*, that is to say 180° must be added$=249°\ 34'\ 07''$—the standard bearing of *A.* from *B.* As it is impossible to set the vernier exactly to this angle seeing that $15''$ is the limit of approximation, the plates must be set in their nearest available position, that is to say $249°\ 34'\ 00''$, which is then tabulated as the first bearing. Proceeding with the observations in precisely the same manner as already explained, the amount of correction to be applied to each bearing from this point is $+10''$ in order to bring the observed bearing *A.* to the standard. Thus the observing of the bearings goes on in the same routine, always choosing for the standard bearing in each case the one which has eminated most directly from the first standard of the survey. For example, in fixing upon the standard at the point *D.*, the bearing *D.A.* should be preferred in place of *D.C.* or *D.B.*, because $D.A.=310°\ 23'\ 55,''$ was the outcome of the first setting, viz. that of *A.*, and is therefore manifestly more reliable than any of the others. This principle should be applied in all angular observations throughout the survey, either in triangulation or traverse. The observations having been completed from all the "Trig" stations, the surveyor now proceeds to compute the angles. This may be done by constructing a rough sketch of the whole triangulation, and marking thereon the observed bearings as corrected in their proper positions. Fig. 21 may be taken as an example. It

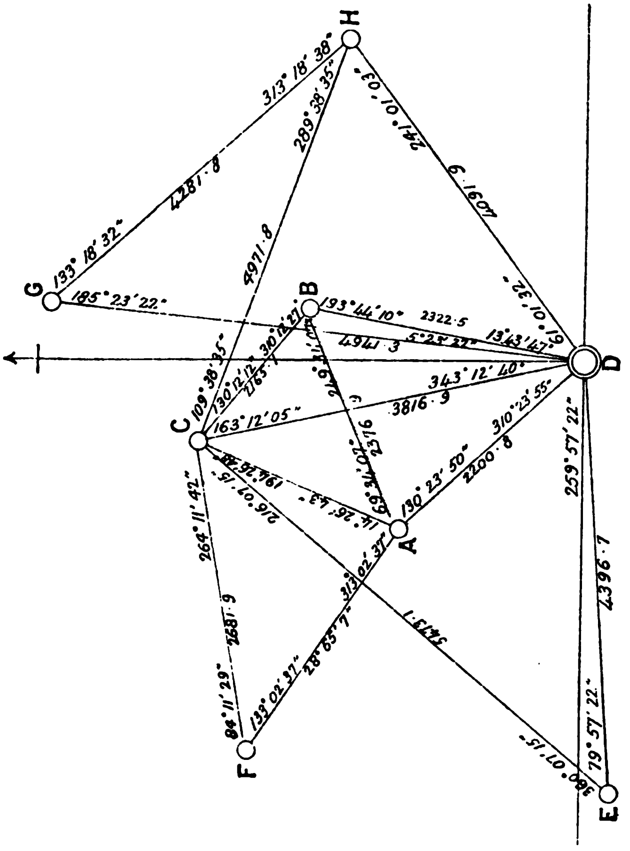


Fig. 21.

now becomes a simple matter of substraction to arrive at the angles, thus the angle *C.A.B.* = 69° 34′ 07″—14° 26′ 45″ = 55° 07′ 22″ as observed. Again, the angle *F.A.C.* = 360° 00′ 00″ —313° 02′ 37″ + 14° 26′ 45″ = 61° 24′ 08″. The difference between this case and the former one arises because the meridian line lies within the angle; the simpler method is to add 360° to 14° 26′ 45″ = 374° 26′ 45″, and substract 313° 02′ 37″, which gives the answer at once.

Now, since the sum of the interior angles of a triangle is equal to two right angles, that is 180°, it follows that if the angles in question do not, when added, reach 180°, a correction must be made. Thus in the triangle *A.B.C.* the sum amounts to 180° 00′ 15″, shewing an excess of 15″, and as these angles were observed under similar conditions with respect to the number of observations, the error must be applied in proportion to the magnitude of each angle, thus, taking angle *A.* of the same triangle—

180° 00′ 15″ : 55° 07′ 22″ : : 15″ : 4″ = correction.

and in angles *B* and *C.* 05″ and 06″ respectively are the calculated corrections, the total making up the 15″ of excess. Subtracting these corrected quantities, the angles, as shewn in the example upon the next page are made to sum up 180°. The object of making these corrections is to enable the surveyor to compute the distances of the sides accurately, otherwise discrepancies would creep into the work. In preparing a triangulation plan of a district, it is the observed and not the corrected angles which are placed upon the plan.

The sum of the three angles of an observed triangle should exceed 180° by reason of the spherical excess; but as this is only discernible in a triangle the sides of which extend to 8 or 10 miles, the matter need not be here treated of. In the triangulation here given, several summations shew a deficiency rather than excess, but unless this error should be considerable, it is better to let the angles stand as observed than attempt to "cook" them, which might instead of doing good only turn out disastrous in the end.

THE CALCULATING AND TABULATING OF THE SIDES.

This operation consists in calculating from a given side and two angles of a triangle the remaining sides. The rule is, "As the sine of the angle opposite the given side is to the sine of the angle opposite the required side, so is the given side to the required side." For this purpose it is usual to have resource to logarithmitic calculation, and in "Chambers's Mathematical Tables" may be found perhaps the best arranged assortment of logarithms, etc., at present in circulation. A surveyor should of necessity have a thorough knowledge of the uses and application of logarithms, and should be able to work them out accurately. All calculations, whether logarithmitic or otherwise, should be carefully checked by different parties. A complete mastery of logarithms enables the student to extend their application, and also to reduce the labour involved in calculating the same question by the ordinary arithmetical process. The following is an example of the method of calculating the sides. In the triangle *A.B.C.* it is required to calculate the side *A.C.* from the given base *A. B.* Then from the above rule—

$$\begin{array}{rr}
\text{As sine of } C. = 64^\circ\ 14'\ 27'' = & 9{\cdot}9545458 \\
\text{is to ,, of } B. = 60^\circ\ 38'\ 15'' = & 9{\cdot}9402847 \\
\text{so is dist. } A.B. = \quad 2376{\cdot}9 = & \underline{3{\cdot}3760109} \\
 & 13{\cdot}3162956 \\
 & 3{\cdot}3617498 = 2300{\cdot}1 = A.C.
\end{array}$$

In this manner the lengths of all the sides may be obtained and marked upon the rough sketch for future use. The surveyor having now got all the angles and sides in the triangulation completed, is in a position to lay these down permanently upon the plan. There are at least three different ways by which this may be accomplished:—

First, By protracting.

Second, By laying down one of the sides in its assumed cardinal direction upon the paper, and from it sweeping the other points with a draughtsman's compass.

Third, By adopting the meridian and perpendicular system.

The *first* method consists in having a circle drawn upon the plan, the circumference of which is divided into degrees, and the centre accurately fixed. Any one of these degree marks may be chosen as the zero or north point, but as surveys are generally made to face the north, the mark nearest the top of the paper is the one adopted as 360° (see Fig. 22. below). Every tenth degree round to the right may then be numbered

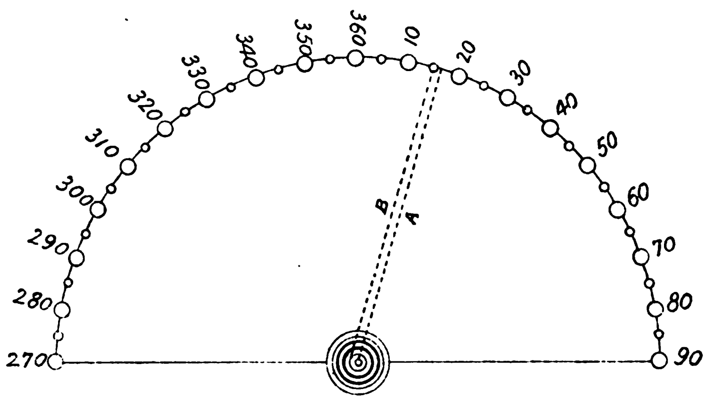


Fig. 22.

for assistance in using the protractor. Now it will be at once apparent that this circle or protractor is exactly similar in its graduations to the limb of the theodolite, therefore a bearing of 20° by the instrument will be represented in direction by a line drawn from the centre *c.* to the point marked 20° upon the circumference in the figure; and, by making use of parallel rulers this bearing may be carried to any part of the paper and applied to any point thereon. It is necessary, however, to provide for the computing of minutes as well as degrees by the protractor, and this is accomplished by means of a simple arrangement of small centre circles. The radius of the largest centre circle is made equal to the length of 1° upon the circumference of the protractor, and dividing this radius into six equal parts, diminishing circles may be drawn. Now, if one end of a parallel ruler is held firm to a certain point upon the circumference of the protractor, while the other is held tangentially to one of these centre circles, it is evident that, upon still continuing to hold the first end in the same position, while moving the other until it becomes a tangent to the next centre circle, the parallel ruler has been caused to move through an arc of one-sixth of a degree, that is to say, 10′. This 10′, being the limit of direct reading power, it follows that the minutes lying between the centre circles must be estimated by the eye. In the figure, the bearing shewn by the dotted line *A.* is 15° 20′. The method of laying this down is to place one end of the ruler upon 15°, and then move the other until 20′ can be read from the centre of the circle, as shewn by the dotted line *B.*; therefore *A.* is the position which the same bearing would occupy, held at the centre and striking the circumference, if ruled forward to its true position. The larger the diameter of a protractor, the greater will be its accuracy; consequently, in constructing one, it will be found that a greater radius can be had upon the sheet of paper by making the protractor a semicircle only. This entails the application of the converse reading, say in all southern bearings. A protractor of this description, with a radius of 18 inches, will be found more reliable, and much handier than the usual small brass ones. After having constructed an accurate protractor, it may be applied to other sheets of paper by pricking through the points with a needle. Protracting is very much practised among that class of surveyors who sacrifice accuracy for the sake of speed; but while it enables survey lines to be laid down with a tolerable degree of accuracy, it appears absurd to lay down bearings which have been observed to seconds, by an agency which can hardly guarantee the result to a minute.

The *second* method is, by applying a small hand protractor to an assumed meridian line, and laying off one of the sides of a triangle by protracting its bearing, and measuring off its length with a scale. From each extremity of this scaled line, sweep with the compasses the other two sides, and their intersection will be the third point. This is a better system than the first, as the angles, being fixed by the scaled lengths of the sides, give a greater assurance of accuracy, and are altogether independent of the errors in construction which protractors are liable to possess. The only fault this method has is, that errors are not got rid of, but are carried forward, and increase as the work goes on. A combination of the first and second methods, however, would enable the operator to have a check upon the bearings laid down, which would go far towards a better result.

The *third* method has the advantage of being subject to the rules of trigonometry, and, not depending upon the accuracy of a protractor, may be truly considered the best. For example, suppose it is required to plot the triangle *A.B.C.* in its true position with relation to the true meridian, calculate the meridian and perpendicular distance of the base line *A.B.*, that is to say, taking *A.* as a fixed point, find the exact distance north and east to *B.* Now the bearing of *B.* from *A.* = 69° 34′ 07″ and its direct distance as chained was found to be 2376·9 (Art. IV. p. 47), so that this line is the hypotenuse of a right-angled triangle, of which the distances to be calculated form the base and perpendicular. For example, in Fig. 23 *S.N.* represents the meridian line drawn through *A*, and *A.B.* the given base. Then *A.x.* will be the distance north, and *x.B.* the distance east to *B.* from *A.* Thus *x.B.* and *x.A.* are respectively the sine and cosine of the angle *N.A.B.*, and the results worked out by calculation shew the position of *B.* as 829·74 north, and 2227·37 east from point *A.* By using a scale and parallel rulers, these distances may be laid off perpendicular to each other in their true positions, from whence the point *B.* is marked, and (by way of check) the scale may be applied to the line *A.B.* to find if it reads the exact chained distance. In this way triangles may be plotted with great accuracy, but where there are a number of them upon a plan, connected, and upon the same standard bearing, the best method of laying them down is to reduce them all to one initial point. The following illustrates what is here meant:—The meridian and perpendicular of *C.* from *B.* is found to be 1397·58 north, and 1653·61 west; thus, adding this northing to the northing of *B.* from *A.*, and subtracting the westing of *C.* from *B.* from the easting of *B.*

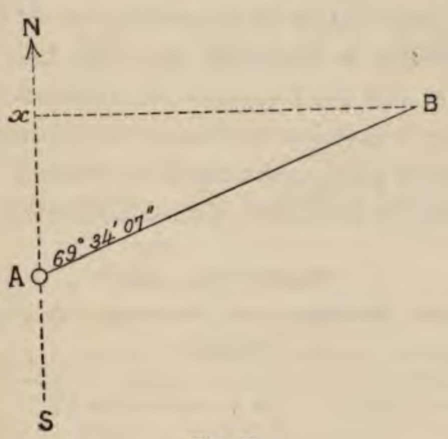


Fig. 23.

from *A*., the result will be the true position of *C*. with relation to the initial point *A*. by direct calculation, which should be equal to the meridian and perpendicular of *A*. from *C*. For example :—

A. to *B*. = 829·74 north	and 2227·37 east.	
B. to *C*. = 1397·58 north	and 1653·61 west.	
Sum 2227·32 north	diff. 573·76 east.	
A. from *C*. 2227·38 south	and 573·79 west.	
diff. 0·06 = error south,	diff. 0·03 = error west.	

As a fuller explanation of the working out of the meridian and perpendicular is afterwards given in Art. VIII., it is unnecessary at this point to carry the examples further; but the above may be sufficient to convince the student that this is the best system of plotting accurate measurements. The following tables shew the usual form of recording these results :—

TABLE No. III.

Triangulation—Meridian and Perpendicular of each Trig Distance.

	On Meridian.		On Perpendicular.		Remarks.
C.D.E.	N.	S.	E.	W.	
C.	4421·04	...	3226·34	...	
D.	...	3654·11	1102·82	...	
E.	...	766·79	...	4329·32	
	4421·04	4420·90	4329·16	4329·32	Error 0·95N. 0·16W.
G.H.D.					
G.	4919·45	...	464·11	...	
H.	...	2937·07	3115·68	...	
D.	...	1982·46	...	3579·6	
	4919·45	4919·53	3579·79	3579·6	Error 0·8S. 0·73E.
C.B.A.					
C.	2227·38	...	573·79	...	
B.	...	1397·58	1653·61	...	
A.	...	829·74	...	2227·37	
	2227·38	2227·32	2227·40	2227·37	Error 0·6N. 0·3E.

TABLE III.—(*continued.*)

A.C.F.	On Meridian.		On Perpendicular.		Remarks.
	N.	S.	E.	W.	
A.	...	1956·0	2094·35	...	
C.	2227·38	...	573·79	...	
F.	...	271·35	...	2668·14	
	2227·38	2227·35	2668·14	2668·14	Error 0·3N.
B.D.A.					
B.	829·74	...	2227·37	...	
D.	...	2256·11	...	551·35	
A.	1426·34	...	...	1676·03	
	2256·08	2256·11	2227·37	2227·38	Error 0·3S. 0·1W.
D.H.C.					
D.	...	3654·11	1102·82	...	
H.	1982·46	...	3579·6	...	
C.	1671·32	...	...	4682·47	
	3653·78	3654·11	4682·42	4682·47	Error 0·33S. 0·5W.
D.B.C.	...				
D.	...	3654·11	1102·82	...	
B.	2256·11	...	551·35	...	
C.	1397·58	...	...	1653·61	
	3653·69	3654·11	1654·17	1653·61	Error 0·42S. 0·56E.
D.A.C.					
D.	...	3654·11	1102·82	...	
A.	1426·34	...	...	1676·03	
C.	2227·38	...	573·79	...	
	3653·72	3654·11	1676·61	1676·03	Error 0·39S. 0·58E.

Total Meridian and Perpendicular from Trig D.

	On Meridian.		On Perpendicular.		Remarks.
	N.	S.	E.	W.	
H.	1982·5	...	3579·6	...	
G.	2936·9	...	...	3115·5	
F.	...	1537·1	...	4234·5	
E.	...	4149·1	...	558·9	
D.	766·8	...	4329·3	...	
	5686·2	5686·2	7908·9	7908·9	

VI. THE TRAVERSE, AND ITS TREATMENT IN THE FIELD.

The traverse, in trigonometrical surveying, is understood to mean the chainage and observations of those lines surveyed for the purpose of fixing the positions of the various fences, roads, streams, etc., or any other objects within or around the area surveyed. Thus, upon the Plan, the traverse lines are those marked by the Roman numeral letters I. II. III. etc., and drawn in red so as to distinguish them from the base line, triangulation lines, check-bearing lines (as XI. to XIII.) or connection line as VIII. to *H.*, or XXI. to *E.* The traverse, therefore, as a whole, means the series of lines surveyed for the purpose of filling in the details. Although it has already been indicated, and will be more clearly demonstrated further on in this work, that through the triangulation the surveyor has a complete means of checking the accuracy of the traverse, it is well in order to reduce the necessary corrections, to endeavour to execute the traversing according to the best practical and mathematical principles. Experience shews that it is entirely impossible to attain perfection in this matter; whether the lines in a closing traverse be numerous and long, or the reverse, there must be a closing error of some sort. Should the closing of a traverse happen to come out exact, it is entirely owing to a chance coincidence, which in all probability may be traced to the balancing of existing equal and opposite errors. Examples of this are by no means rare, and miles of traverse have been executed without the application of checks, and yet the closing bearing found to read upon the limb of the instrument precisely as it originally stood in the field book. It is almost unnecessary to remark that a traverse turning out like this ought to be repeated, as it is perfectly within the range of possibility that two serious errors may have occurred, which, by counterbalancing each other, have caused this wonderful result, and escaped detection.

Before beginning to traverse, it is necessary to *set out* the poles, as at I. II. III., etc. (upon the Plan). This preliminary of setting out must be done with considerable judgment, because a deal of extra and needless work may be saved by exerting a little forethought upon the matter. For instance, the following precepts may be taken into account:—

1. The longer the lines, the better.
2. The shorter the offsets, the better.
3. The more check-bearings, the better.
4. The flatter the ground chained over, the better.
5. The firmer the legs of the theodolite can be planted, the better.

Circumstances may arise requiring other considerations to be taken into account, but if a traverse is set out in conformity with the above, it will be found to give satisfaction in the economising of time, and the fostering of accuracy.

Having thus set out as many of the traverse rods as possible, plant the theodolite over the "Trig" station from which it is intended to connect the traverse, adjusting and clamping the lower plate upon the triangulation standard bearing. The Plan shews trig *D.* as the point at which the survey is commenced, and for reasons already given (Art. V. page 60), *D.A.* is the standard bearing to which the instrument is set = 343° 12′ 40″. Release the upper plate, and turning the instrument round to the right, observe traverse station I. = 98° 05′ 00″. Then turn the instrument round in the same direction until the cross hairs again coincide with Trig *A.*, and notice if the reading upon the plate is the same as before. If, for instance, it should happen to read a minute more or less, apply this to the former bearing, writing the minutes underneath in the field book; repeat the operation completely again, noting down the precise reading upon I., made subject to the closing reading, and accept the mean as the true bearing of I. It is better never to leave a traverse station without reading all the bearings taken from it, at least twice, and the back or closing bearing three times. By carrying out this rule, a traverse of considerable length may be observed with a great assurance of accuracy.

The chainmen now proceed with the measuring of the connection line *D.* to *I.*, which should be done twice over, as it, not being part of the main traverse should carry as little error with it as possible, for reasons afterwards pointed out. Offsets to the sides of the river should be taken where bends occur, and particular attention paid to the side which forms the boundary of the survey.

Station I. having been reached, the instrument is adjusted over the hole, and the limb set to the converse mean reading of the back bearing = 278° 05′; from this point, however, the bearing to XXIX. = 35° 49′ 30″ is also taken, as by doing so now, the necessity is obviated of again having to set up the instrument at this station. After what has already been written, it will be unnecessary to go over an explanation again of the routine of the instrument work, but it may be repeated that the instrument must always be set upon the converse reading of the back bearing, so that the forward bearing (in the order of the traverse) indicates the true bearing in the direction surveyed.

Upon reaching station V., XXIX. = 281° 49′ 15″, is also read and recorded in the field book. At VIII. a connection is effected with Trig *H.*, and, being carefully re-measured, affords not only a complete check to bearing VIII. to *H.*, but also to the actual distance arrived at through the reduction of the traverse by logarithmitic calculation of the length *H.D.* Setting up at Trig *H.*, and referring to the Table, *H.* to *D.*, is manifestly the standard which should be adopted = 241° 01′ 03″. In thus setting, afterwards releasing the upper plate, and turning the instrument on VIII., the difference between this and the previous reading from VIII. may be found, this difference is called the angular error. Note down the new reading immediately above the original one, provided it is reasonably near, that is to say, within 3′. As the new bearing depended directly upon a standard, it must be adopted, and the error thrown upon the traverse. The usual way of applying the correction is to divide the error by the number of lines in the traverse, and add or subtract the results (as the case may be) in a regularly decreasing ratio backwards upon the lines of the traverse, as far as the error is admissible. This system of distribution would be perfectly correct, provided the lines so treated were of an equal length, upon the assumption that the

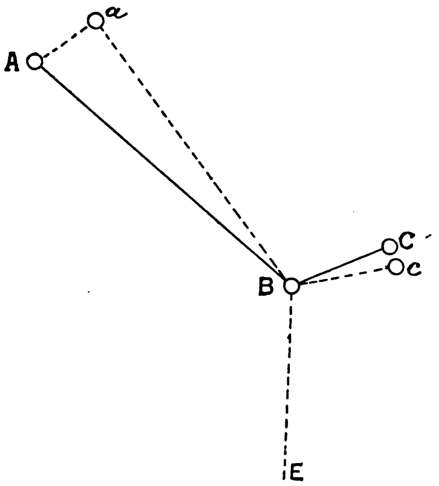


Fig. 24.

error is progressive. It is, however, found from practice, that the error itself has been contracted, in a great measure, through the lines not being of equal length, consequently the natural way of distributing the error among the lines, is to do so with reference to their lengths. For example, let the two lines *A.B.* and *B.C.*, Fig. 26, be of unequal length, and *B.E.* a standard bearing line; and suppose an error of two minutes has been indicated, let the lines *B.a.* and *B.c.* represent the true position which the traverse should occupy. It is obvious that the error *A.a.* must be greater than *C.c.*, because the radius is greater; consequently it follows that an angular error has a greater effect upon a long line than a short one, and therefore it is not right to apply the same amount of correction to both lines.

To shew the method of applying the angular correction to a traverse, let the following Fig. 25 represent a portion of a traverse started at I. set on *A.* standard = 220° 15′. Upon reaching station V., the bearing to VI. reads 48° 57′; as VI. to *B* = 2° 38′ is also a standard bearing, the proper method of obtaining the check upon planting at VI. is, to set this standard upon the limb, and clamping on *B.* turn the instrument round upon V., so that the error may be shewn upon the last traverse line. Having done this, let it be supposed that the check is found to be 6′ less than the original, that is to say, 51′ instead of 57′; take care to insert the bearing in the field-book as shewn, writing the letter *c.* (for "correct") after it for identification. The surveyor has now to consider whether this error of 6′ has arisen gradually, or accumulated at one place. A glance at the figure will shew that with reference to what has already been said, a great deal of the error is likely to have been contracted through inaccurate manipulation at the point IV., as III. to IV. is only 250 links in length, whereas IV. to V. is 750. In this case it will be safer to apply a greater proportion of the correction to the line III. to IV. than to any of the others. But as bearing I. to II. has been observed directly from the standard *A*., it should not be altered; consequently only the three lines in the centre have to be dealt with. Proceed therefore by deducting 6′ from the bearing V. to VI., 5′ from bearing IV. to V. = 114° 29′, giving a total correction of one minute for angle IV., V., VI. Then deduct 2′ from bearing III. to IV. = 7° 10′—the true bearing; so that the total correction is for 3′ for angle III., IV., V. Lastly, 1′ deducted from bearing II. to III. = 101° 42′, the true bearing of line II. to III., with a total correction of 1′ for angle II., III., IV. Had the three centre lines been nearly equal in length, the adjustment would have consisted simply in subtracting 6′, 4′, 2′ and 1′ backwards upon the traverse. Seeing that so much error is liable to arise from the use of a short line, it is better to have a check-bearing applied to one of its extremities, so that the instrument may be set upon the bearing instead of the short one; for example, let Fig. 26 represent a series of lines. At station II. observe IV. as well as III., then proceed to III., set on II., and observe

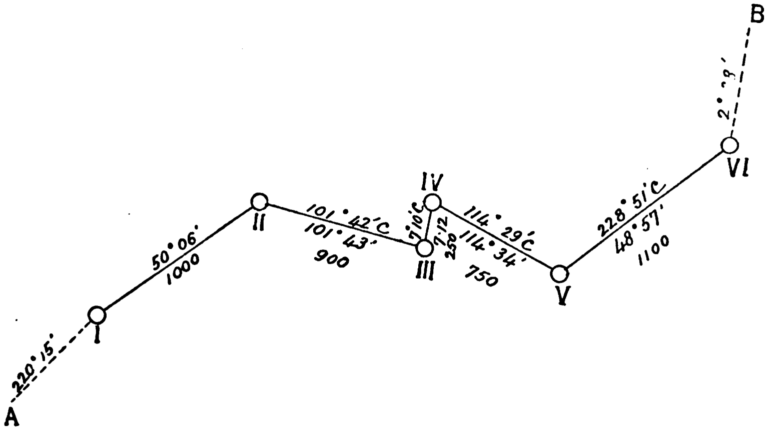


Fig. 25.

IV. afterwards at IV., set on II., observe V. and III. The difference of previous reading will be found, the mean of which

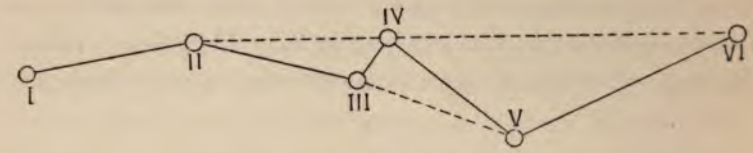


Fig. 26.

will be the true bearing of line III. to IV. Upon closing at VI., after taking these precautions the error may not exceed 1′. If IV. cannot be seen from II., it may be possible to see V. from III., in which case, at III. set on II. and observe IV. and V., then at IV. set on III. and observe V.; at V. set on III. and observe IV. This last bearing will be the correct one, and half its difference from the original must be applied to the line III. to IV. plus or minus as the case may be.

It is a habit with surveyors, in executing a zig-zag traverse like the above, when two of the next forward stations are visible from one point, to proceed after the following fashion:—Plant the instrument at II. set on I. and observe IV. and III., then proceed direct to IV. set on II. and observe VI. V. and III., and lastly at VI. set on IV. and observe V. By this means the time and trouble of planting at III. and V. are avoided; but it is easy to shew that this system, although handy, is altogether inaccurate. Two triangles, it will be noticed, have been enclosed without the possibility of the surveyor ascertaining the amount of the closing error in each, and in plotting the work by the protractor (which is the favourite method with this class of surveyors), an error is very likely discovered, the blame of which is entirely laid upon the shoulders of the chainmen, the surveyor considering himself justified in doing so, because his bearings closed correctly at VI. A good way of curing a surveyor of this habit would be to shew him the precise errors in figures by reduction to the meridian and perpendicular, and afterwards proceed to III. and V. with the instrument so as to shew the cause thereof. Surveyors who employ this " handy way of getting over a traverse" often consider themselves smart in the field, and never pause to reflect whether the system they are about to apply is good or bad, provided the ground is got over quickly.

In traversing short lines, the extremities of which can only be seen from pole to pole, it is advisable to apply check bearings, otherwise errors will creep into the work. These errors have their origin in defective adjustments of the instrument, or, from the fact of the strong magnifying power of the telescope, rendering it difficult to cause the cross hairs to exactly cut the centre of the rod, because if the centre is not bisected, the actual deviation from the true centre is magnified laterally. Another frequent cause of angular error is the deflections of the rods from the true perpendicular, especially where the tops only can be seen. The best way of getting over these difficulties

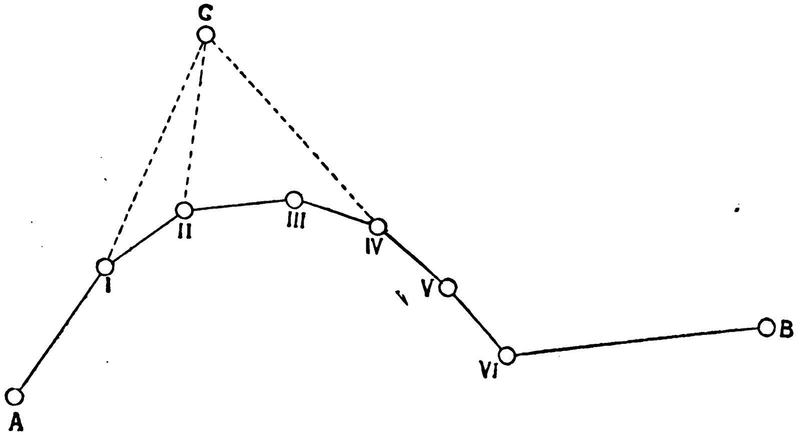


Fig. 27.

is to sight to a plumb-bob string held up exactly over the station hole, which gives a finer point for observation than a magnified pole. Fig. 27 is intended to represent a traverse of short lines. By fixing any point *C.* at a distance outside of the traverse, from which one or more of the forward points are visible, check bearings, as *C.*I., *C.*II., and *C.*IV., may be obtained, which have a tendency to keep the bearings of the traverse within a reasonable limit of error. The method of procedure is—plant on I., set on standard bearing *A.* (meanwhile sending an assistant to stick up a pole at point *C.*), observe *C.* and II. Proceed direct to *C.* set on I., and observe II., and IV., then return to the traverse, planting at *A.*, set on *C.*, observe bearing I. to II., correcting if necessary. Working forward until IV. is reached, again set on *C.* and apply correction to bearing III. to IV. This operation necessitates a little extra work in the field, but this it will always repay. Should an observation be taken to the top of a pole afterwards found not to be standing perpendicular, the easiest way of getting over the difficulty is to suspend a plumb-bob from the observed part of the pole and reset the pole to the position of the plumb-string, obliterating the old mark in the ground, and adjusting the chainage measurement to the new. It is always advisable to complete a traverse, or at least a connection with some given point, in finishing a day's work in the field. If possible, the rods of a traverse should be left standing in their stations until its accuracy has been proved, and before a pole is lifted, the station should be well marked by cutting trenches, placing three stones round it, or driving in a substantial peg into the hole. Any check pole from which other points have been observed should be left standing until these points are reached with the instrument.

In traversing a series of lines which run nearly in the same course, it is sometimes difficult in sighting backwards to single out the exact pole from which the check-bearing had been taken. In such circumstances, it is better to make use of a pole with a distinguishing coloured flag. The best way of applying a check to a traverse of this description is to run out a chain-line from it, nearly at right angles to the general route, and observing its extremity from any point upon the traverse, a check will be given to bearings and distance by calculation from this measured line. Care must be taken to make the line of sufficient length to cause the angles to be well-conditioned. Fig. 28 may be taken as an example of a traverse I. to VIII., and II. to *A*., the chained and observed check line.

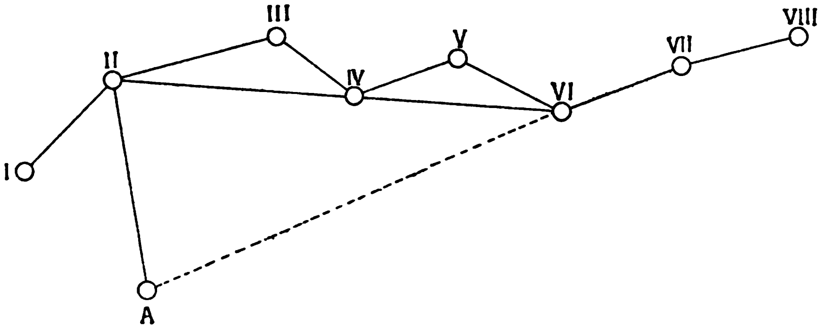


Fig. 28.

Upon planting at II., III. VI. and *A*. are observed; then plant at *A*. and observe VI.; proceed again with the traverse from III. until VI. is reached, and bearing V. to VI. checked. The distance II. to VI. may also be calculated from the base II. to *A*., so that if the traversing has been done accurately between II. and VI., the total meridian and perpendicular distances of each line between II. and VI. should equal the meridian and perpendicular of the calculated distance II. to VI. In cases where a chain line of a traverse must pass over rough or steep ground, the best way of ascertaining its correct measurement is to run out a base from one of its extremities upon flatter ground, of a suitable length, so that the three extremities of the triangle thus found are visible from each other. This operation may be treated upon the same principles as are applied to triangulation; but, generally speaking, one observation only will be found necessary at each point. Sometimes a part of the traverse itself may serve as a base from which to calculate the required measurement.

The example given upon the plan is of this nature, XXXV. to *C.* is the calculated line, as taken from the traverse base XXXIV. to XXXV. This system of calculating from a base carefully chained over moderately flat ground, is invaluable to a surveyor, because by its application nearly all the difficulties arising through inaccessible distances may be surmounted. At any rate, a surveyor, practising this system, will seldom find it necessary to rack his brains for a solution amongst the multitude of problems set forth upon the subject. Whenever a very rough line presents itself, instead of attempting to use the chain, it will be found better in the long run to adopt the above method. When it is required to take offsets from a line thus calculated, it is imperative that it should be chained; but the calculated distance should be accepted as the total measurement of the line.

In laying out a traverse upon the field, it is often convenient, and even necessary, to produce a line over a hill; this is termed ranging. There are two methods of ranging a line: the first is ranging by the eye, and the second, ranging by the theodolite. The principles of ranging are based upon the following self-evident fact: If, in the whole straight line *A.B.*

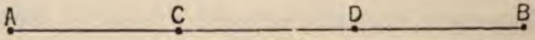


Fig. 29.

(Fig. 29), the points *C.* and *D.* are coincident, as seen from the extremities, then, from point *C.* the part *C.D.* will be visibly in the same straight line with *C.B.* Again, from point *D.* the part *D.C.* will also be visibly in the same straight line with *D.A.*, so that *C.* and *D.* must be intermediates upon the line *A.B.*, even although *B.* is not visible from *A.* Suppose, however, *A.* and *B.* to be visible from each other, all that is required to be done is simply to motion an assistant into line so that he may fix in poles at *C.* or *D.*; he will then be enabled to check the fixing of the second rod by lining himself with the first, afterwards, again proceeding to the first, and checking its position by the second. In this way a line may be ranged with considerable accuracy by the eye. In a case where *A.* and *B.* are not visible from each other, proceed to a point upon the line in the supposed direction from which *A.* and *B.* are visible, and instruct an assistant to go in the direction of *B.*, as far as he can, without losing sight of *A.*; then call upon the assistant standing near *D.* to range his pole so that the point you occupy near *C.* is intermediate with *A.* Then observe where the assistant's pole stands with relation to *B.*, and if it is not intermediate, shift your pole in the direction the assistant is required to move; call upon him again to take up the line, and if necessary, proceed to shift first your own pole, and then the assistant's, until the latter, after shifting, is truly intermediate with *B.* Then fix in the poles permanently, thereby indicating the true line. Points thus ranged should never be sighted to or from by the theodolite, as their exactitude cannot be relied upon to such a degree as to warrant their acceptance for the angular observations of an accurate survey. Lines ranged in this fashion may be found perfectly suitable in the working of a chain survey, as a means for keeping the chaining in line from which offsets are to be taken.

In a survey, on the other hand, the theodolite is always used for fixing intermediate points, and ranging out or producing lines, so that the second method, viz., ranging by theodolite, comes next under consideration.

Referring again to the example in Fig. 29, and taking the case where *A.* and *B.* are visible from each other, the method of ranging the intermediate points with the theodolite is as follows:—Set up the instrument, for example, at *A.*, adjust it by exercising the greatest possible care in the process of levelling, clamp the plates firmly together, and directing the telescope to *B.*, cause the intersection of the hairs to coincide, by using the lower plate and vertical arc clamp and tangent screws. Then, looking along the top of the telescope, signal one of the rods into the field of view, using the tangent screw of the vertical arc if a gradient is found to intervene. Motion the rod gradually towards the intersections, then again use the tangent screw of the vertical arc, so as to bring the intersection upwards or downwards to the foot of the pole, signal again as before, and whenever the coincidence exists, cause the assistant to stick in the pole. Look through the telescope to see that the pole is right after it is stuck in, then unclamp the vertical arc if necessary, and bring the cross hairs back to *A.* If the result is satisfactory, motion to the chainman, so that he may proceed to the next point. The assistant should never leave a point until (upon the surveyor having checked it and found it correct) a signal to this effect is given.

Suppose *A.B.*, Fig. 30, to represent a line running up the face of a hill with *B.* as the summit, and from this point it is

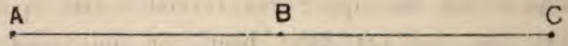


Fig. 30.

required to produce *B. C.* in the same straight line with *A.B.*

There are various ways of doing this by the theodolite; and as it is sometimes necessary, and very often convenient, to carry out this operation in a traverse, it may be well to consider the matter in detail. Thus, in setting up and adjusting the instrument at *B.* set on *A.*, the following methods may be pursued, viz.:

(1) By setting the instrument on a back or any bearing in the usual way, and turning round the upper plate through 180°, setting pole *C.*, and checking back again upon *A.*

(2) By proceeding as in the first case, but checking back by undoing the clips, and turning the telescope end for end, observing if the cross-hairs still intersect at the point *A.*

(3) By clamping the upper and lower plates on *A.* without any reference to bearings at all; undoing the telescope and placing it end for end, checking back by lifting the telescope into its former position, observing if the cross-hairs still intersect at the point *A.*

(4) By proceeding as in the last case, but with this difference, —that in placing the telescope end for end, it is turned half round upon its own axis, checking back upon *A.* by lifting the telescope into its former position.

In method (1), the accuracy of the result depends upon three assumptions—First, that the lower plate has not shifted after being clamped, and the upper one turned round upon it; second, that the verniers have been set and read with exactitude; and third, that the collimation line of the instrument is in perfect adjustment in its lateral position.

In method (2) it is obvious that the same three assumptions must be accepted, but it is possible that the error of graduation may be added to the error of collimation without detection.

In method (3) it will be apparent that there can be only one cause of error, and that is with respect to the adjustment of the line of collimation. And it will be seen that in thus discarding the use of the graduated arc, no error is accumulated; the whole operation simply consisting in the producing of a line, the bearing of which has previously been observed from point *A*.

In order to mark the effect of a collimation error, take the following examples—Fig 31. Let *B.* represent a horizontal section or plan of a telescope, of which *e.* is the eye-piece, and *e.A.* the line of collimation given by an instrument in error; and let *e.C.* be the true line. In the second case of the same Fig., the telescope is supposed to have been turned end for end, as in method (3); consequently the lines of true and false collimation are reversed. Now let Fig. 32 below represent the telescope *B.* with these lines drawn in different directions.

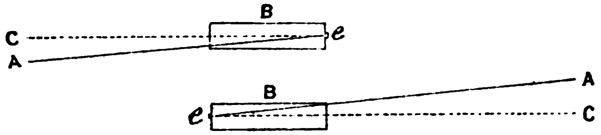


Fig. 31.

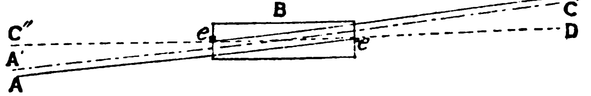


Fig. 32.

From this it will be seen that the false collimation line *A.e.* is not in one continuous straight line with *e.C.*, but is broken at both ends of the telescope, through the existence of an error of collimation. In adopting method (4), this error is entirely obviated by turning the telescope about its own axis also, which gives the straight line *A′.C′.* with all accuracy—*C″.D′.* being the position which the true line should occupy: its difference may be corrected with reference to adjustments (Art. III). The surveyor, in producing lines, should bear in mind what has already been pointed out in this Art., page 73, regarding the disadvantage of setting off a long bearing from a short one.

The following case, Fig. 33, very often occurs in practice:

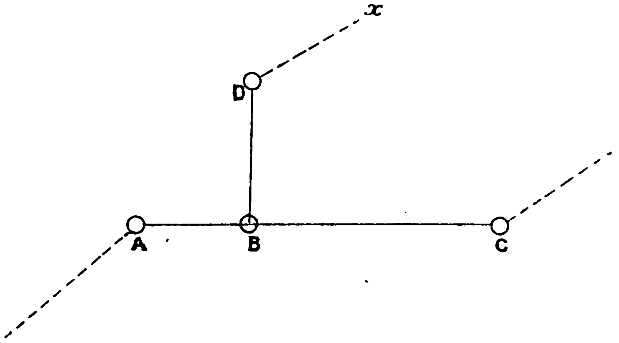


Fig. 33.

A.C. represents one continuous line having an intermediate point *B.* from which the the traverse *B.D.x.* has afterwards to be run. Thus, in setting up at *B.* to observe *D.*, instead of clamping the instrument upon *A.*, it should be clamped upon *C.*, the more distant point.

LAYING OFF ROADS.

In feuing districts and in new countries, the laying out of new roads form a great part of the field operations of a surveyor. At first sight, and to any one imperfectly acquainted with the art of surveying, it would naturally appear a simple process to lay off the road *A.B.*, Fig. 34., by merely traversing one side, *A.b.c.B.*, and laying down the other side *d.e.f.g.* upon the ground by ranging parallel lines at an equal width, and putting in corner pegs at their intersections *e.f.* This no doubt is the natural way, and indeed the method almost universally adopted in this country; but however simple, it is greatly subject to error, and certainly cannot furnish mathematical accuracy. The defects of this system may be more clearly demonstrated by comparing it with another and better method, which is as follows:—

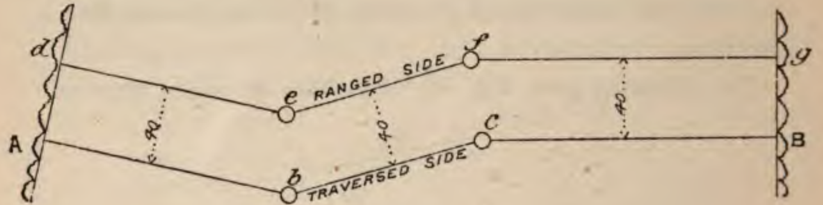
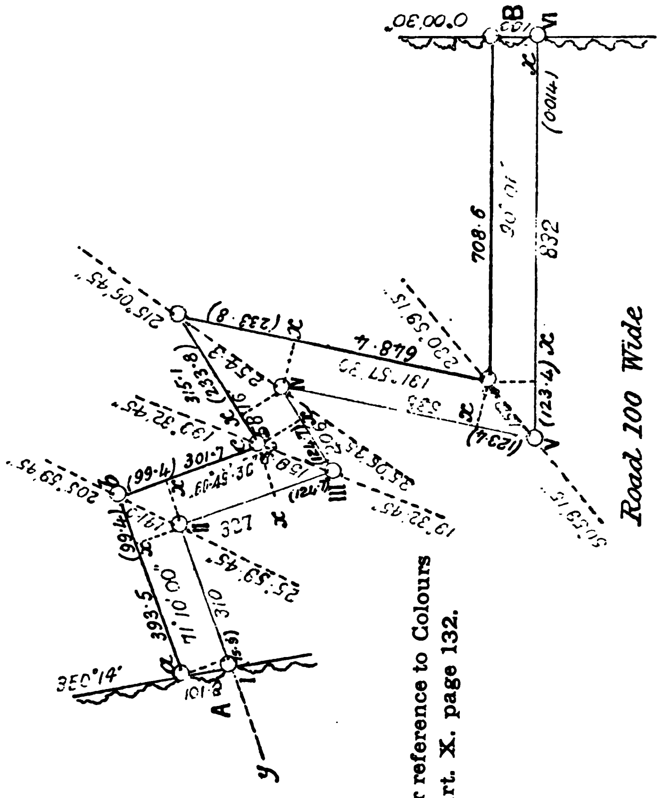


Fig. 34.

Having fixed upon the starting-point, and terminus of a road, and also its general route, the surveyor adopts one side of the supposed road for his traverse, and at every bend-point, rendered necessary by the nature of the ground or route, the bearings are observed in the usual manner.

Thus in Fig 35, it is proposed to lay off the road *A.B.* one chain in width. The traverse is started at peg I., or upon the right hand side of the road looking forward. The instrument being set on the fence bearing=350° 14′, and II. observed =271° 10′, it is obvious that in order to fix the opposite side



NOTE—For reference to Colours
See Art. X. page 132.

Fig. 35.

it is necessary to calculate and lay off distance I.*a*. This may be done by the following calculations:—

At I.

By outside angle.

° ′
I. to *a*. = 350 14
Converse I.-II. = 251 10
99 04 = angle *y*.I.*a*.
− 90 00
I.*a*.*x*. = sec. 9 04 = 10·0054604
log width = 100 = 2·0000000
tan. = 9·2029714
12·0054604 = 101·27 = I.*a*.
11·2029714 = 15·96 = I.*x*.

By inside angle.

° ′
Cardinal 360 00
I. to *a*. − 350 14
9 46
I. to II. + 71 10
Angle *x*.I.*a*. = cosec 80 56 = 10·0054604
log. width = 100 = 2·0000000
Cotan = 9·2029714
12·0054604 = 101·27 = I.*a*.
11·2029714 = 15·96 = I.*x*.

From the above it will be seen that the quantity I.*x*., which represents the exact distance of increase or decrease in length of the opposite side, may be readily calculated. The calculated distance I.*a*. is then laid off with the chain, and marked by driving a peg into the ground at *A*. These calculated distances must be marked or noted down in the field-book, as they will afterwards be referred to.

Proceeding to II., it becomes necessary to calculate the bearing to *B*. as well as the distances *x*.*B*.*B*. II. The following shews how this is accomplished:—

At II.

By outside angle.

° ′ ″
Converse I. II. = 251 10 00
,, II. III. − 160 49 30
= 90 20 30 = angle I. II. III. outside.
÷ 2
= 45 10 15 = ½ angle
+ II. III. = 160 49 30
205 59 45 = converse
− 180 00 00
25 59 45 = bearing of II.-*b*.

Therefore—

½ angle cosec 45° 10′ 15″ = 10·1492240
log 100 = 2·0000000
cotan = 9·9974103
12·1492240 = 141·00 = II.*b*.
11·9974103 = 99·41 = *b*.*x*.

By inside angle.

° ′ ″
Cardinal 360 00 00
Converse I. II. − 251 10 00
108 50 00
+ II. II. = 160 49 30
269 39 30 = angle I. II. III. inside.
÷ 2
= 134 49 45 = ½ angle.
160 49 30
25 59 45 = true bearing II.*b*.

Therefore—

° ′ ″
½ angle 134 49 45
− 90 00 00
sec 44 49 45 = 10·1492240
log 100 = 2·0000000
tan = 9·9974103
12·1492240 = 141·00 = II.*c*.
11·9974103 = 99·41 = II.*x*.

The instrument is then set on bearing 25° 59′ 45″, and the point laid down at the calculated distance 141·00. The following shows the calculations at each point onwards:—

At III.

By outside angle.

	° ′ ″	
	340 49 30	
	58 16 00	
	282 33 30	= angle II. III. IV. outside.
÷ 2		
	141 16 45	= ½ angle
III. IV. +	58 16 00	
	199 32 45	
	- 180 00 00	
	19 32 45	= true bearing III.*c*.

Therefore—

	° ′ ″	
½ angle	141 16 45	
	- 90 00 00	
sec	51 16 45	= 10·2037542
	log 100	= 2·0000000
tan		= 10·0959620
		12·2037542 = 159·87 = III.*c*.
		12·0959620 = 124·73 = III.*x*.

By inside angle.

	° ′ ″	
	360 00 00	
III. IV. +	58 16 00	
	418 16 00	
Converse II. III.	- 340 49 30	
	77 26 30	= angle II. III. IV. inside.
÷ 2		
	38 43 15	= ½ angle.
III. IV.	58 16 00	
	19 32 45	= true bearing III.-*c*.

Therefore—

$$\begin{array}{rcl} \text{cosec } 38^\circ\ 43'\ 15'' & = & 10{\cdot}2037542 \\ \log\ 100 & = & 2{\cdot}0000000 \\ \text{cotan} & = & 10{\cdot}0959620 \\ \hline & & 12{\cdot}2037542 = 159{\cdot}87 = \text{III.}c. \\ & & 12{\cdot}0959620 = 124{\cdot}73 = \text{III.}x. \end{array}$$

At IV.

By outside angle.

$$\begin{array}{lrrrl} & ^\circ & ' & '' & \\ & 238 & 16 & 00 & \\ & 191 & 57 & 30 & \\ \hline & 46 & 18 & 30 & \\ \div 2 & & & & \\ \hline & 23 & 09 & 15 & = \tfrac{1}{2}\text{ angle.} \\ + & 191 & 57 & 30 & \\ \hline & 215 & 06 & 45 & \\ - & 180 & 00 & 00 & \\ \hline & 35 & 06 & 45 & = \text{true bearing.} \\ \hline \end{array}$$

Therefore—

$$\begin{array}{rcl} \tfrac{1}{2}\text{ angle cosec } 23^\circ\ 09'\ 15'' & = & 10{\cdot}4053793 \\ \log\ 100 & = & 2{\cdot}0000000 \\ \text{cotan} & = & 10{\cdot}3689075 \\ \hline & & 12{\cdot}4053793 = 254{\cdot}32 = \text{IV.}d. \\ & & 12{\cdot}3689075 = 233{\cdot}83 = d.x. \end{array}$$

By inside angle.

$$\begin{array}{lrrrl} & ^\circ & ' & '' & \\ & 360 & 00 & 00 & \\ & 238 & 16 & 00 & \\ \hline & 121 & 44 & 00 & \\ & 191 & 57 & 30 & \\ \hline & 313 & 41 & 30 & \\ \div 2 & & & & \\ \hline & 156 & 50 & 45 & = \tfrac{1}{2}\text{ angle.} \\ & 191 & 57 & 30 & \\ \hline & 35 & 06 & 45 & = \text{true bearing.} \\ \hline \end{array}$$

Therefore

° ′ ″
½ angle = 156 50 45
− 90 00 00
sec 66 50 45 = 10·4053793
log 100 = 2·0000000
tan = 10·3689075
12·4053793 = 254·32 = IV. *d*
12·3689075 = 233·83 = *d. x.*

At V.

By outside angle.

° ′ ″
360 00 00
11 57 30 converse
371 57 30
− 90 01 00
281 56 30
÷ 2
140 58 15
90 01 00
230 59 15
− 180 00 00
50 59 15 = true bearing.

Therefore

° ′ ″
½ angle = 140 58 15
− 90 00 00
sec 50 58 15 = 10·2008553
log 100 = 2·0000000
tan = 10·0911787
12·2008553 = 158·80 = *V. e.*
12·0911787 = 123·36 = *e.x.*

By inside angle.

° ′ ″
90 01 00
11 57 30
78 03 30
÷ 2
39 01 45
90 01 00
50 59 15 = true bearing.

Therefore—

½ angle cosec = 39° 01′ 45″ = 10·2008553
log. 100 = 2·0000000
cotan = 10·0911787

12·2008553 = 158·80 = V.*e*.
12·0911787 = 123·36 = *e.x.*

The last point on the traverse is treated in a manner similar to the starting point, thus:—

At VI.

By outside angle.

° ′ ″
Converse V. VI. = 270 01 00
Converse fence bearing = 180 00 30

90 00 30
− 90 00 00

sec 0 00 30 = 10·0000000
log. 100 = 2·0000000
tan 6·1626962

2·0000000 = 100·00 = VI.*f*.
8·1626962 = 0·01 = VI.*x*.

By inside angle.

° ′ ″
Fence = 360 00 30
Converse V. VI. = 270 01 00

cosec 89 59 30 = 10·0000000
log. 100 = 2·0000000
cotan = 6·1626962

2·0000000 = 100·00 = VI.*f*.
8·1626962 = 0·01 = VI.*x*.

In each case both examples have been given in detail, because, as the cases continually change with every alteration in the direction of the road line, it is safer to use both the outside and inside angles as a check; these calculations having to be worked out in the field, often under unfavourable circumstances, it is easy to fall into error in laying off the pegs, and unless it is of such extent as to be noticeable by the eye, it cannot be detected except by repetition of the work or actual traverse of the other side of the road.

Now supposing it is found more convenient to traverse the left side of the same road, Fig. 36 and its accompanying calculations will show the difference by comparison. At I. the operation will be precisely the same as before, and as in all the cases the results will be similar, only one example need be given for each, thus:—

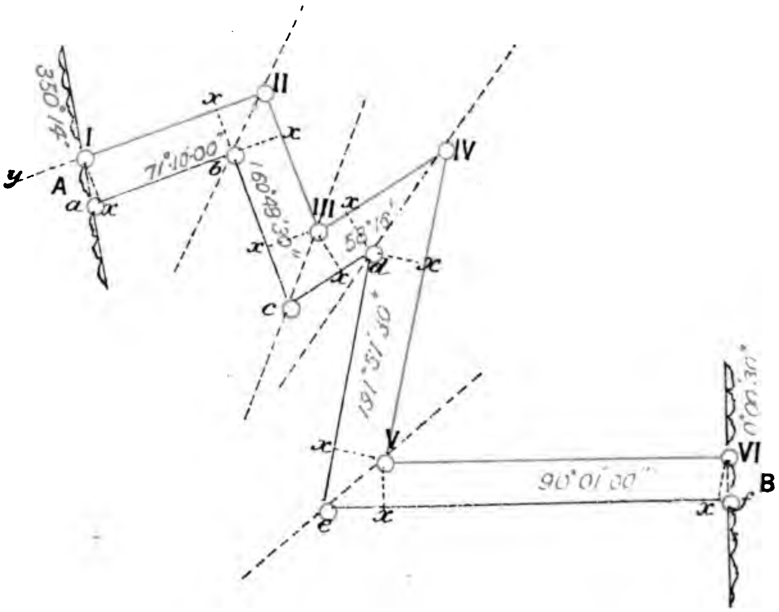


Fig. 36.

At II.

By inside angle

	°	′	″	
I. II. converse	251	10	00	
	160	49	30	
	90	20	30	= inside angle.
÷ 2				
	45	10	15	= ½ angle.
II. III. +	160	49	30	
	205	59	45	= true bearing II.*b*.

Therefore—

½ angle cosec = 45° 10′ 15″ = 10·1492240
log. 100 = 2·0000000
cotan = 9·9974103
12·1492240 = 141·00 = II.*b*.
11·9974103 = 99·41 = *b.x*.

There is another way of manipulating the angles so as to get the diagonal bearing, and as this may be applied to the examples which follow, it is better that it should be explained here. In

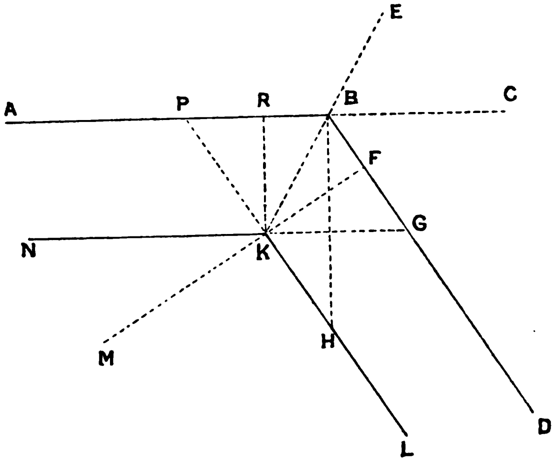


Fig. 37.

Fig. 37 let *A.B.D.* and *N.K.L.* represent the parallel sides of a road:—the angle of deflection *C.B.D.* = *R.K.F.* = twice the angle from which the results are calculated viz. *R.K.B.* or *B.K.F.*

> *Proof*:—Because angle *C.B.E.* + *E.B.K.* + *R.B.H.* = *F.K.L.* + *L.K.M.* + *M.K.P.*, each being together equal to three right angles, and the angle *H.B.G.* = *P.K.R.*; therefore the remaining angle *C.B.D.* = the remaining angle *R.K.F.*

From this take the following rule:—"Find the deflection angle by subtracting the one bearing from the other, half of which will be the angle adjacent to the radius, then to the forward bearing add 90° minus the angle already found, the remainder will be the true bearing of the diagonal."

Applying this system we have the following:—thus

At II.

$$\begin{array}{rrrrl}
 & {}^\circ & {}' & {}'' & \\
\text{II. III.} = & 160 & 49 & 30 & \\
 & 71 & 10 & 00 & \\
\cline{2-4}
 & 89 & 39 & 30 & \\
\div 2 & & & & \\
\cline{2-4}
x.b.\text{II.} = \text{sec.} = & 44 & 49 & 45 & = 10{\cdot}1492240 \\
\text{log.} & 100 & & & = 2{\cdot}0000000 \\
\text{tan.} & & & & = 9{\cdot}9974103 \\
\cline{5-5}
 & & & & 12{\cdot}1492240 = 141{\cdot}00 = \text{II.}e. \\
 & & & & 11{\cdot}9974103 = 99{\cdot}41 = \text{II.}x.
\end{array}$$

for the diagonal bearing—

$$\begin{array}{rrrrl}
 & {}^\circ & {}' & {}'' & \\
\text{bearing II. } b = & 160 & 49 & 30 & \\
+ & 90 & 00 & 00 & \\
\cline{2-4}
 & 250 & 49 & 30 & \\
x.b.\text{II.} \quad - & 44 & 49 & 45 & \\
\cline{2-4}
 & 205 & 59 & 45 & = \text{true bearing II.}b.
\end{array}$$

AT III.

	°	′	″	
II. III.	160	49	30	
III. IV.	58	16	00	
÷2				
	102	33	30	= deflection
x.III.c. = sec. =	51	16	45	= 10·2037542
log.	100			= 2·0000000
tan.				= 10·0959620

12·2037542 = 159·87 = III.c.
12·0959620 = 124·73 = III.x.

For the diagonal bearing.

	°	′	″	
	58	16	00	= forward bearing.
	+90	00	00	
x.III.c.	+51	16	45	= ½ angle.
	199	32	45	= bearing III.c.

Note.—Case III. differs slightly from the former, owing to the fact that the bend in the road is in the opposite direction; so that the right angle and half angle are both added to the forward bearing to give III.c. Thus, when the bend is to the right, add 90° minus the half angle, but when it is to the left, add 90° plus the half angle. By keeping this rule in mind, the surveyor need never be at a loss as to procedure.

AT IV.

	°	′	″	
IV. V. =	191	57	30	
III. IV. −	58	16	00	
÷ 2				
	132	41	30	= deflection.
x.IV.d. = sec =	66	50	45	= 10·4053793
	log. 100			= 2·0000000
tan.				= 10·3689075

12·4053793 = 254·32 = IV.d.
12·3689075 = 233·83 = d.x.

For the diagonal bearing.

	°	′	″	
	191	57	30	= forward bearing.
+	90	00	00	
	281	57	30	
x.IV.d.	66	50	45	= ½ angle.
	215	06	45	= bearing IV.d.

AT V.

° ′ ″
IV. V. 191 57 30
V. VI. − 90 01 00
101 56 30
÷ 2
x.V.*e*. = sec = 50 58 15 = 10·2008553
log. 100 = 2·0000000
tan = 10·0911787
12·2008553 = 158·80 = V.*e*.
12·0911787 = 123·36 = *e*.*x*.

For the diagonal bearing.
° ′ ″
90 01 00
+ 90 00 00
+ 50 58 15 = *x*.V.*e*.
230 59 15 = bearing V.*e*.

At VI. the operation is precisely the same as already shewn. In traversing the right hand side of a road, the working out of the deflection angle is exactly the converse, so that it is needless to give detailed examples thereof.

Some surveyors make use of the interior and exterior angle system, while others use the deflection angles; but upon the whole, it is better to be thoroughly conversant with both methods, as each result may be checked by a different calculation. In some cases one may be abandoned in favour of the other, if found more expeditious, when applied to a certain case. Generally speaking, however, the deflection angle system will be found to have the advantage over the others with respect to speed; and in this system also, the trigonometrical conditions are always uniform.

The following figures shew how the various distances are found which compose the opposite side of a road, where the right hand side is traversed (see Fig. 35).

Distance I. II. = 310 − 15·9 + 99·4 = 393·5 = *a.b.*
,, II. III. = 327 + 99·4 − 124·7 = 301·7 = *b.c.*
,, III. IV. = 206 − 124·7 + 233·8 = 315·1 = *c.d.*
,, IV. V. = 538 − 123·4 + 233·8 = 648·4 = *d.e.*
,, V. VI. = 832 − 123·4 − 0·01 = 708·6 = *e.f.*

Sometimes in traversing and laying off a road, it is necessary or convenient to change the traverse across from the one side of the road to the other; and reference to Fig. 38 will

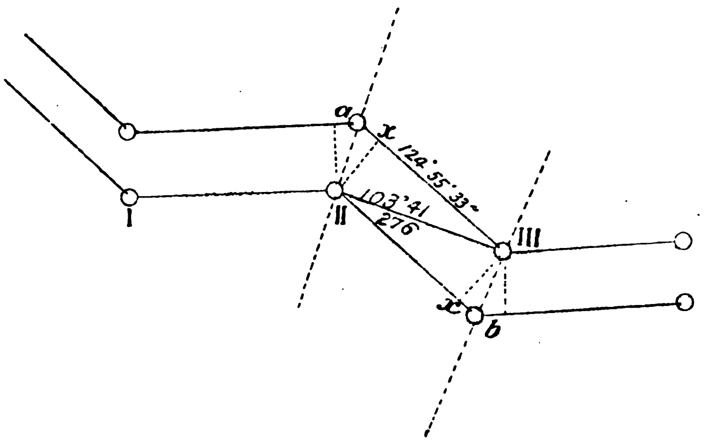


Fig. 38.

serve to illustrate how this may be accurately done, without thereby weakening the traverse. Let it be required to change over from peg II. to III, proceed thus:—

Selecting III. as a suitable point to change over to; from II. set on I. observe III. = 103° 41′ and chain the distance = 276: from this data, calculate bearing II.*b*. or III.*a*., by finding angle *x*.II. III. or *x*.III. II., thus:—

Length log. = 276 = 2·4409091
Width log. = 100 = 2·0000000
cosec 10·4409091 = 21° 14′ 33″ = *x*.II. III. or *x*.III. II.

Therefore—

$$\begin{array}{rrrrl} & ^{\circ} & ' & '' & \\ & 103 & 41 & 00 & = \text{II. III.} \\ + & 21 & 14 & 33 & \\ \hline & 124 & 55 & 33 & = \text{bearing II.}b.\text{ or III.}a. \end{array}$$

The position of the corner peg *a*. may then be calculated with respect to this bearing and the foregoing one on the traverse, and set off before the instrument is removed from II. Proceeding to III. and setting back upon II., a check bearing may be taken to *a.*, which should read the converse of the calculated one=304° 55′ 33″. The traverse is then proceeded with in the usual way.

At points where one road branches off from another, a considerable amount of calculation is necessary for putting in the junction pegs; but as the conditions are so varied as to render it unsuitable here to give examples of every possible case, only one will be taken, which occurs frequently in practice, and is troublesome to deal with. Thus:—From a main road *A.B.* (Fig. 39), one chain in width, it is proposed to set off a junction with a branch road *c.*, 60 links wide, starting from opposite peg X. It will be seen that although the diagonal peg *B.* of the main road, does not require to be fixed permanently, the two starting pegs *a.* and *c.* must be put in, and all the distances of the sides ascertained, as *f.a.*, *a.g.*, *e.c.*, and *c.I.*

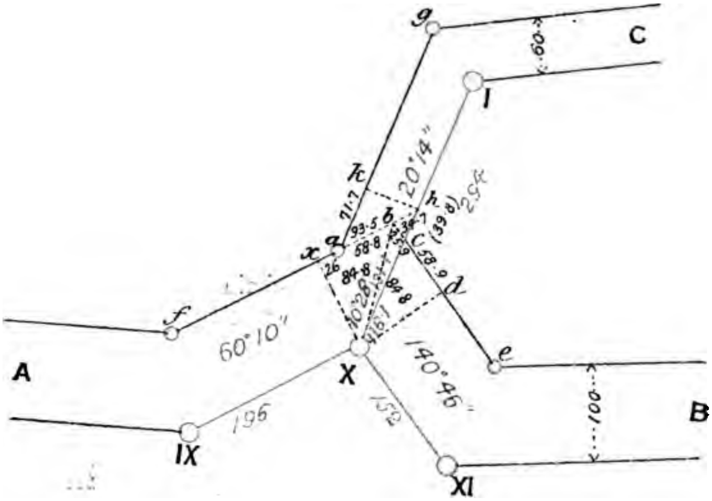


Fig. 39.

Suppose the theodolite to be stationed at X., and bearings X. I. and X. XI. observed, proceed to find the bearing and distance to *b.*, and also distances *b.x.* and *b.d.* in the usual manner, thus :—

° ′
140 46
60 10
80 36 = angle of deflection
40 18 = angle *b.x.d.*

° ′
140 46
− 90 00
50 46 = bearing X.
− 40 18 = angle *b.x.d.*
10 28 = bearing X.*b.*

sec. 40° 18′ = 10·1176643
log. 100 = 2·0000000
tan. = 9·9284274
12·1176643 = 131·1 = X.*b.*
11·9284274 = 84·8 = *b.*X. or *b.d.*

Bearing X.*d.* = 50° 46′
" X. I. = 20° 14′
30° 32′ = angle *c.*X.*d.*

sec. = 30° 32′ = 10·0648285
100 = 2·0000000
tan. = 9·7707261
12·0648285 = 116·1 = X.*c.*
11·7707261 = 58·9 = *c.d.*

so that *b.d.* = 84·8
c.d. = − 58·9
25·9 = *c.b.*

In order to calculate the corresponding lengths, find the deflection angle of the branch road, viz., the angle *b.a.g.*, thus :—

cosec 39° 56′ = 10·1925354
log 60 = 1·7781513
cotan = 10·0772129
11·9706867 = 93·5 = *a.h.*
11·8553642 = 71·7 = *a.k.*

To find *x.a.*, it is necessary first to calculate the distance *h.b.*, and as *c.h.* is also required, it may be obtained from the same base *b.c.*, thus angle *b.h.c* = *b.a.k.* = 39° 56′ = the deflection angle, and angle $b.c.h.$ = the complement of angle $c.X.d.$ = 30° 32′, therefore $b.c.h.$ = 59° 28′: this may also be found by taking the

converse bearing X.I. = 200° 14′
,, $b.c.$ – 140° 46′
59° 28′ = $b.c.h.$

but the angle $e.b.h.$, = the deflection angle at peg X., so that these three together should equal two right angles:—

	°	′
angle $h.$ =	39	56
,, $c.$ =	59	28
,, $b.$ =	80	36
	180	00

Making use of these angles, therefore, we have the following:—

sine $h.$ = 39° 56′ = 9·8074646
sine $c.$ = 59° 28′ = 9·9351715
log. 25·9 = 1·4132998
11·3484713
1·5410067 = 34·7 = $b.h.$

sine $h.$ = 39° 56′ = 9·8074646
sine $b.$ = 80° 36 = 9·9941289
log. 25·9 = 1·4132998
11·4074287
1·5999641 = 39·8 = $c.h.$

so that the whole $a.h.$ = 93·5
minus the part $b.h.$ = 34·7
58·8 = $a.b.$

and the whole $x.b.$ = 84·8
minus the part $a.b.$ = 58·8
26·0 = $x.a.$

These calculations completed, the points $c.$ and $h.$ may be set off in the bearing X.-I., the theodolite having been left adjusted at X. for that purpose; then planting at $h.$, set on X., observe $f.$, thereby obtaining an angular check; and also check the position of $b.$ with respect to its being intermediate with the line $h.f.$ Test the length $h.b.$, and from $b.$ set off the distance $b.a.$ The position of $c.$ may be checked after the diagonal point $e.$ has been put in by planting at $e.$ or $b.$, and testing whether it is intermediate upon that line. Pegs are then driven in at $a.$ etc., and the remaining distances of opposite sides of the roads, viz., $f.x.$, $k.g.$, and $d.e.$, calculated with respect to their deflection angles as already given in detail. Problems like these will appear to many a waste of time, but strict adherence to figures is the only way to attain mathematical accuracy; and to attempt, in such cases, to range in the points by setting off equal widths

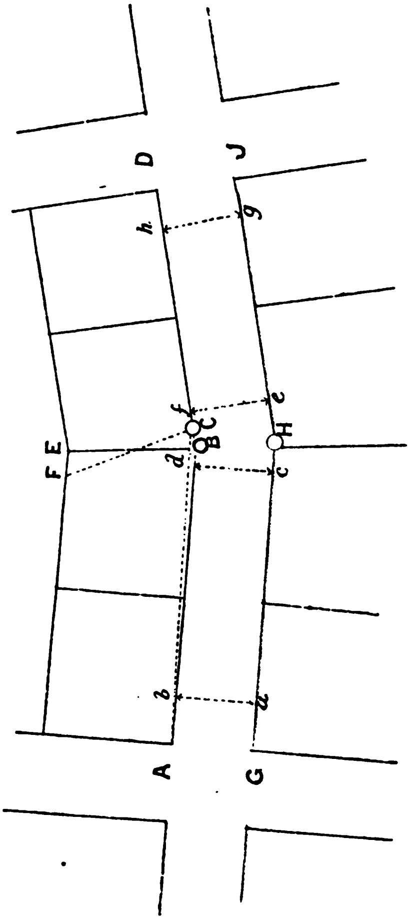


Fig. 40.

(thereafter having to scale a drawing to find the lengths of the other sides) occupies quite as much time, and is attended by such inconvenience, as to completely do away with any advantage which simplicity of working out may give; and most important of all, errors peculiar to mechanical construction creep in, and render the work so defective as to be practically useless. Take for example a case upon feuing ground of valuable frontage. The following Fig. 40 may be taken as an illustration of the kind of error which is sometimes made in laying off roads upon feuing ground by the common system of ranging out the other side of the road.

Let *G.H.J.* be the traversed side of a main road, from which it is intended to lay of parallel lines *A.B.B.D.* by equal widths *a.b.*, *c.d.*, *e.f.*, and *g.h.*, in order to find the intersection or corner peg *B.* Now, in taking these measurements of widths, it is obvious that an error, however slight, must materially affect the position of *B.*, so that for example, if the width *c.d.* be taken "full," point *c.* may represent the corner. A glance at the Fig. will shew that in the event of the road being formed and fenced with *c.* as the corner, and also that feus I. and IV. have been taken up and built upon, a serious difficulty may arise,—for while II. and III. have been purchased with respect to the equal areas thereof, as given upon the Feuing Plan, it is found that in giving possession, and allotting to each feuar the area paid for, an excambion has to be effected, by shifting the boundary line to *C.F.* And even this is done to the detriment of Feu III., by reducing its frontage. In the case of a turn in the road, having a slight deflection angle of say 3°, an error of half a foot in a width would cause the intersecting point to be situated 9 feet from its true position. Although this is an extreme case, a little carelessness on the part of an assistant might cause a blunder of equal, or even greater extent.

VII. OFFSETS AND FIELD-BOOK.

Offsets are measurements taken at right angles to a chain-line to objects near, but not coincident therewith, either for the sake of finding the area of the space defined by these objects, or for the purpose of fixing the positions of the objects themselves.

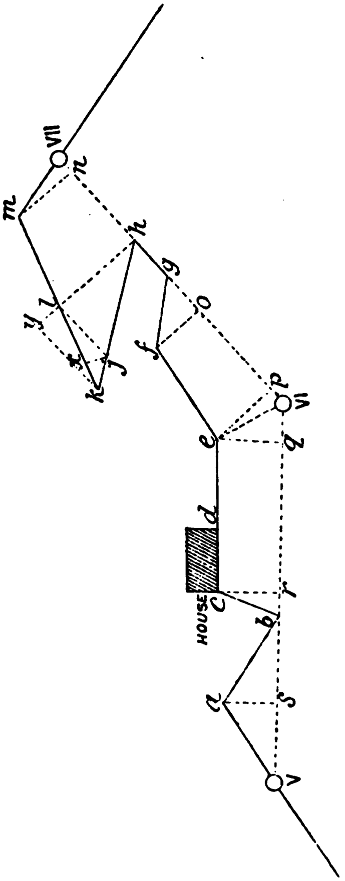


Fig. 41.

In Fig. 41 the traverse is represented by IV., V., VI. and

VII., and from these chain lines it is required to fix the various objects, as indicated by the letters from *a.* to *m.*, in order that these points may be delineated upon the Plan, as well as to ascertain the areas enclosed. In the first triangle V.*a.b.*, it is obvious that *a.* might be fixed by measuring V.*a.* as a production of IV. V., and afterwards by measuring V.*b.*, the lines V.*a.a.b* would be fixed; but in order to obtain the area therefrom, a trigonometrical calculation would be necessary. Again, by measuring V.*b.* and *s.a.*, the area would be easily obtained, thus —distance $V.b. \times \frac{1}{2}\, s.a.$:—but in this case, on the other hand, the point *a.* would not be fixed. To meet both requirements, therefore, the simplest way is to set off and measure the perpendicular offset *s.a.*, noting down the length upon the chain line to *s.* from V., thereafter chaining onwards to *b.*, and noting also the length V.*b.*

In plotting the figure, the distance to *a.* is scaled at right angles to the chain line from point *s.*, and in calculating the area of the triangle V.*a.b.*, the operation is simply to multiply the whole base V.*b.* by half the perpendicular *s.a.*, as above, or *vice versa*.

The position of *c.* is fixed by noting distances V.*r.* and *r.c.*, so that $V.r. - V.b. = b.r. \times \frac{1}{2}\, r.c. =$ area *r.b.c.* The position of the house may be fixed from line *c.e.*, and its dimensions measured, and marked in the field-book, as shewn; point *e.* is fixed by offset and distance in the same way, but in order to arrive at the area *c.e.q.r.*, the distance *q.r.* must be multiplied by the mean or average of the two offsets *r.c.* and *q.e.* Arriving at peg VI. the whole length V. VI. is entered in the field-book. Now, were it only required to fix the boundary line, the survey might at once proceed as far as *o.* measuring *o.f.*; but in order to find the area, another offset must be taken to point *e.* from *p.*, at right angles to the new chain line VI. VII., so that the two triangles thus formed, viz., *q.e.*VI., and VI.*e.p.*, may be readily calculated along with the other figures $p.o.f.e$ and $o.g.f.$ Assuming that point $k.$ cannot be reached by a direct offset from the chain line, the method shown in the fig. may be adopted. From $h.$ the offset $h.l.$ is taken, and from $l.$ the distance $l.j.$, and $l.k.$ and $x.j.$ at right angles therefrom. From this the area $h.k.l.$ may be calculated, thus:—$h.l. \times \frac{1}{2} l.j. + l.k. \times \frac{1}{2} x.j.$ A shorter method would be to measure $h.l.$, and $l.j.$, as formerly, producing $h.l.$ to $y.$, measuring $k.y.$ also at right angles and parallel to $j.l.$ The area $h.k.l.$ would therefore be the mean proportional of the two triangles $l.j.h.$ and $y.k.h.$ In the first method, the point $k.$ may be fixed by producing $m.l.$ to the distance $l.k.$, and by the last method by the measured offset from y; the adoption of either of these methods is entirely governed by circumstances. The remaining portions of the example need not be here detailed, as their treatment is precisely the same as that already explained. Offsets should not exceed 150 links in length, and must be set off from the chain line with great accuracy by means of the optical square.

In town surveying, where the greater part of the work consists in offsetting, each offset should be tied by two other distances, forming a well-conditioned triangle, of which the offset is the perpendicular; and between the points so fixed a dimension line should be measured as an extra check (see Fig. 42). Great care requires to be taken in offsetting a crooked boundary, especially where the bends are gradual. In a case like this, it will often become advisable to take two distances

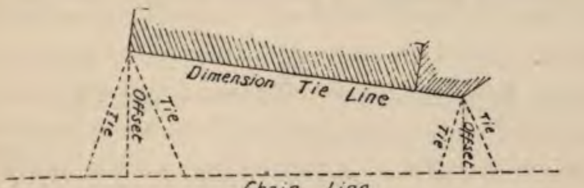


Fig. 42.

IV. THE BA

BE

As the base l
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from each point upon the chain line, the one to be used in obtaining the area, the other in the plotting; for example Fig. 43. As x is the point required to be fixed, the triangle abc would manifestly be smaller than the correct area of the enclosed ground, so that by producing b.c. to d, the portion of

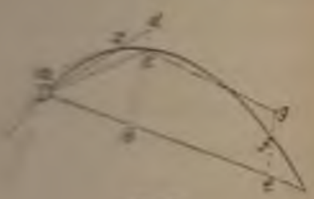

Fig. 43.

ground cut off inside between a. and x. would be equal to the portion cut off outside between x. and d, then the base abd would equal the true area; the same method may be carried out from e. This system of "giving and taking" may be done with considerable accuracy by the eye, but it should only be carried out when the parcels of areas are insignificant in extent, and must be regulated with respect to the length between offset points. Offsets should never be measured to a straight part of a boundary between bends, because the slightest deflection of the chain from the true line or an inaccurately measured offset may cause a bend to appear upon the plan at a point where the boundary is perfectly straight upon the ground,

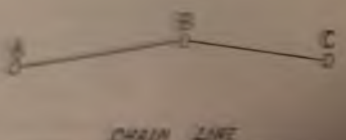

Fig. 44.

and likewise also [illegible] *already offsetted, th* tion in which the survey is progressing, in order to nt where the next offset will be required. Fig. 44 importance of this. Suppose *A.* the point just off- looking forward from *A.* towards *C.*, it will be easier the existence of a bend at *B.* than if the observer is at *B.* itself; in fact, unless the deflection is considerable, almost impossible to make certain that a bend really Generally speaking, the correctness of offsets depends e following conditions:—

hat the tape line used is in correct adjustment to the rd measure.

That the offset chain line is perfectly straight.

That the optical square is in adjustment.

That the point noted down as the distance upon the chain- , (from which the square is taken) is exactly that over which optical square is held when observing.

5. That in measuring off the length the tape is held to the act point to be fixed, and the measurement taken close to e ground.

THE FIELD BOOK.

Surveyors should cultivate a clean and neat method of recording field measurements. The notes should be made so that others not actually engaged in the field work may understand them; one of the best, although perhaps the oldest method, is simply to draw a sketch of the surrounding objects shewing all the chain lines and observed bearing lines in their true positions, writing thereon all distances, b[illegible] offset-lengths, remarks, etc., in conjuncti[illegible] the [illegible] ns. Care should be taken not to en[illegible] re there is not sufficien[illegible] e for [illegible] carried forward [illegible] fresh page. Each point of the traverse should be numbered, so that in thus carrying forward from page to page, no difficulty can arise in following the connection. The fact of the field book thus becoming a sort of map of the ground enables the draughtsman intelligently to plot the work, although he may not have seen the ground.

VIII. THE REDUCTION OF THE TRAVERSE TO MERIDIAN AND PERPENDICULAR.

The basis of the meridian and perpendicular system rests upon the self-evident fact, that in traversing from one point to another by a route, or routes, however circuitous, the position of the one point with respect to the other, as calculated from the traversed route, should be identical with the position of the same point as shewn from some other source. To effect this, the true cardinal directions of the lines in the traverse are set against each other in the form of positive and negative quantities. Also, in traversing backwards to the original starting point, whether by the same or some other route, the signs of the quantities only are changed, the results coming out the same as before.

In regard to direction, it may be here noted, referring to Fig. 10, p. 23, that as 360° stands for north, any bearing between that and 90° is considered north and east, between 90° and 180° south and east, between 180° and 270° south and west, and between 270° and 360° north and west. In Fig. 45 let *A.B.* represent two points, their relative positions as in bearing and distance being known. It is required to shew that by reducing the traverse *A.*I., I. II. II.*B.*, the combined results of these lines should be equal to the meridian and perpendicular distances of

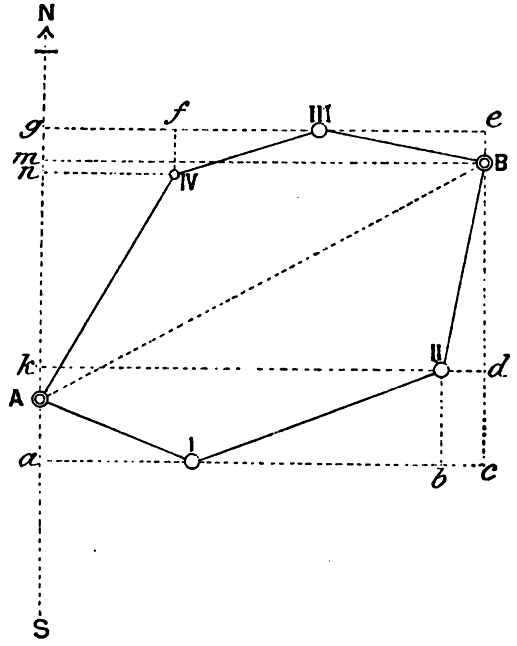

Fig. 45.

A.B. direct. Let the dotted lines outside the traverse represent meridian and perpendicular lines: *N.*, being the north point, *A.*I. runs in a south and east direction, represented by *A.a.* and *a.*I. respectively; I. II. runs in a north and east direction, represented by *b.*II. and *b.*I. respectively, therefore the position of II. with relation to *A.* is—

*A.*I. = *a.a.* south and *a.*I. east.
I.II. = *b.*II. north and I.*b.* east.

by adding like, and subtracting unlike signs, thus:—

$$\left.\begin{array}{l} A.\text{II.} = A.a. - b.\text{II.} = A.k.\ \text{north} \\ \quad ,, \ = a.\text{I.} + \text{I.}b. = k.\text{II.}\ \text{east} \end{array}\right\} = \text{Meridian and perpendicular of II. from } A.$$

Passing onwards to *B.*, the line II.*B.* runs in a north and east direction, so that in order to obtain the check which the previous knowledge of position *B.* affords, the quantities *d.B.* and *d.*II. must also be applied, thus:—

Position of II. = *A.k.* south, and *k.*II. east from *A.*,

so that *B* = *d.b.* north, and II. *d.* east from II.

by adding and subtracting **$d.B. - A.k. = A.m.$ north** } **= Meridian and perpendi-**
and $k.\text{II}. + \text{II}.d. = m.B.$ east } **cular of *A.B.***

By taking *A.m.* and *m.B.* as two sides of a right-angled triangle, not only may the distance *A.B.* be readily calculated, but also its true bearing; and these being entirely the product of the traverse by comparison with the known quantities, its accuracy is proved. It will be seen from the Fig. that in traversing back again to *A.* by III. and IV., or otherwise, the same conditions must exist in a converse form. For example, the total distance *a.c.*, traversed in an easterly direction from *A.* to *B.* must equal the distance *e.g.* traversed in a westerly direction from *B.* to *A.* Similarly also the distance north of *B.* from *A.* must manifestly be equal to the distance south to *A.* from *B.* Now, the following Fig. 46 taken from the Plan, may serve for an example, as to the working out of the traverse reduction. It may be

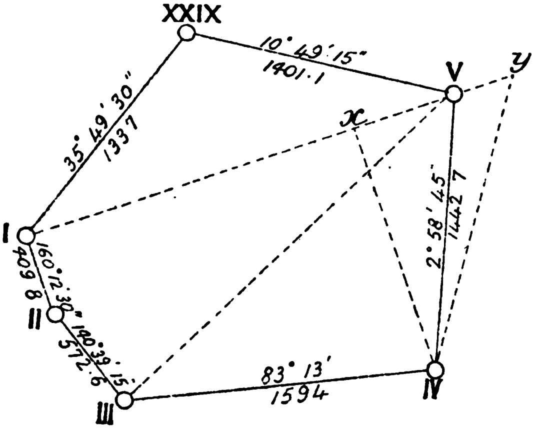

Fig. 46.

noticed in starting, that in the previous example, Fig. 45, the work was "closed upon a given distance and bearing," whereas this traverse about to be considered is what is termed "closed upon itself."

Starting at peg I. the bearings are reduced as they stand in the field-book as far as peg V., but in order to complete the circuit back to I., the remaining two bearings require to be applied in their converse forms, because they were observed in the opposite direction, and marked accordingly in the field notes. The following shews the working out of the meridian and perpendicular of each line in the traverse :—

II.
sine 160° 12′ 30″ = 9·5296883
log. 409·8 = 2·6125720
cos. = 9·9735574
12·1422603 = 138·8 E.
12·8561294 = 385·6 S.

III.
sine 140° 39′ 15″ = 9·8020890
log. 572·6 = 2·7578513
cos. = 9·8883667
12·5599403 = 363·0 E.
12·6462180 = 442·8 S.

IV.
sine 83° 13′ 00″ = 9·9969492
log. 1594 = 3·2024883
cos. = 9·0723055
13·1994375 = 1582·8 E.
12·2747938 = 188·3 N.

V.
sine 2° 58′ 45″ = 8·7157751
log. 1442·7 = 3·1581760
cos. = 9·9994127
11·8739511 = 74·8 E.
13·1575887 = 1437·4 N.

XXIX.
sine 281° 49′ 15″ = 9·3114400
log. 1401·1 = 3.1464691
cos. = 9·9906908
12·4579091 = 287·0 N.
13·1371599 = 1371·4 W.

I.
sine 215° 49′ 30″ = 9·7673871
log. 1337 = 3·1261314
cos. = 9·9089183
12·8935184 = 782·6 W
13·0350497 = 1084·0 S

Having checked these calculations carefully, proceed to tabulate the results in the following form, remembering that if the field work has been done "consistently," the northings and southings should balance.

TABLE No. IV.

	MERIDIAN.		PERPENDICULAR.	
	N.	S.	E.	W.
II.	...	385·6	138·8	...
III.	...	442·8	363·0	...
IV.	188·3	...	1582·8	...
V.	1437·4	...	74·8	...
XXIX.	287·0	...	...	1371·4
I.	...	1084·0	...	782·6
	1912·7	1912·4	2159·4	2154·0
	1912·4	...	2154·0	...
	0·3	...	5·4	...

It will be seen, however, that the quantities do not quite balance; this difference is what is termed the traverse error, and it is to the rectifying of this that the surveyor has now to turn his attention, taking into consideration the following:—

(1). Whether this error is within a reasonable limit, taking the length of the traverse into account, and also the nature of the ground.

(2). Whether the error may have been caused by the instrument.

(3). Whether the error may have been caused by the chain.

As pointed out already in a preceding article, it is practically impossible to do chaining without contracting chainage error, therefore it is usual to fix a limit of error, reckoned at so much per mile, and subject to the general nature of the country. In very rough districts an error of 8 links per mile may be allowed; in others, 6 links, where the ground, although not exactly rough, is undulating; and in flat districts the standard may be fixed at 4 links per mile. In the calculation of trig distances, ranging from 2 to 3 miles, from ¼ to ½ a link per mile, may be taken as a reasonable limit of error, which of course is to be considered due to angular causes. The three angles in a "trig" triangle should not have a greater error than from 30″ to 45.″ The limit of closing error in a traverse may be taken as from 1 to 2 minutes. Thus, then, these rules of error limit should always be considered in order to determine the steps to be taken where correction is necessary. Where the error found in this way composes two sides of a right-angled triangle, the total error as represented by the hypotemuse should be shewn, so that the above error should be taken as 5·5 instead of the figures in the table. This, however, is seldom done in practice.

The results upon the Plan will be found in some instances to exceed the limit of error; but (for the sake of example), a better opportunity is thereby given for the recognizing of such, and the modes of correction may be more clearly shewn.

From what has already been explained, it is reasonable to assume that the above traverse has been defectively chained; because the surrounding work has proved the accuracy of the bearings, and as the traverse is "closed upon itself," there is nothing to determine exactly the position of any point upon it, except its closing in upon itself; so that, had the chainage been done consistently, the error would have come out very much less. For example, if the whole of the traverse had been chained in a uniformly imperfect manner, or by an incorrect chain, the errors in northings and southings, eastings and westings, would still balance each other. Thus, if the chaining gives too much northing from I. to V. by chaining back from V. to I. in the same inaccurate manner, the southing will also be too much; but these quantities balancing, no error is detected; the same applies to the eastings and westings. Had this error been so great as to infer that a blunder had been committed, attributable for instance to the misreading of the chain, the only course is to chain all the lines over again in the opposite direction, and then, failing to find the error, re-observe all the bearings with the theodolite. At first sight this last test may appear absurd, because the bearings were found correct by the closing observation, but it is nevertheless a fact that a mistake in the bearing of a certain line may exist notwithstanding the check. For example, a surveyor may be in the habit of trusting to memory instead of turning up the field-book to find the back bearings, so that if he inadvertently marked down a wrong bearing while he had the correct bearing in his memory, and set the instruments thereon, no blunder would appear when closing in, but the calculations, on the other hand, would shew the error.

Some surveyors, upon finding that the traverse error exceeds the limit, do not hesitate to alter their chainage distance and bearings, by deducting or even adding a link here and there, so as to make the work close. It is hardly necessary to give detailed reasons why this should never be done; it is sufficient that such a procedure is known by the significant term of "cooking," and is certain to be found out in the long run. Such corrections have a bad effect upon chainmen, causing them to become careless in their duties, knowing that their results, however defective, will be accepted, without further trouble on their part. The advice here given is, that if the error is up to, or within the limit, the surveyor should upon no account alter the distances or bearings by "cooking," but should the error exceed the limit, a complete repetition of the field work should be made, both as regards distances and bearings.

In the tabulated example already given, the several results represent the meridian and perpendicular of each line, or, more strictly speaking, of each point with relation to the preceding one. For the future application of these results, it is the rule to reduce them to one datum in such a manner as at once to shew the meridian and perpendicular of each point from this datum. In a large survey the datum point selected is universally a "Trig" station, situated near the centre of the work. The triangles connected therewith should be well conditioned, and the "Trig" station itself placed in a commanding and prominent position. The traverse thus reduced is called the Total Traverse from "Trig" (giving the name of the trig). The first series of results, consisting of the positions from point to point, is called the Single Traverse, of which the preceding table is an example. The following method should be adopted in reducing the whole traverse. Selecting I. as the datum or initial point, construct a set of columns exactly similar to and adjoining those of the single traverse, and begin by inserting therein the position of one as zero, then the meridian and perpendicular of II. are extended into the new columns: proceed by calculating the others, thus:—

II. =	385·6 S.	and	138·8 E.	from I.
III. + =	442·8 S.	„	+ 363·0 E.	
	828·4 S.	„	501·8 E.	= III. from I.
IV. - =	188·3 N.	„	+ 1582·8 E.	
	640·1 S.	„	2884·6 E.	= IV. from I.
V. - =	1437·4 N.	„	+ 74·8 E.	
	797·3 N.	„	2159·4 E.	= V. from I.
XXIX. + =	287·0 N.	„	- 1371·4 W.	
	1084·3 N.	„	788·0 E.	= XXIX. from I.
I. - =	1084·0 S.	„	- 782.6 W.	
Error =	0·3 N.	Error =	5.4 E.	

The following table may be taken as an example of a traverse sheet, constructed somewhat in the form used by the New Zealand Government.

TABLE No. V.

Traverse of Field Boundary marked I., V., XXIX., I.

Page of Field Book.	No. of Peg.	Cardinal Direction.	True Bearing.	Measured Distance.	Single Traverse.				Total Traverse.				Remarks.
					Meridian.		Perpendicular.		Meridian		Perpendicular.		
					N.	S.	E.	W.	N.	S.	E.	W.	
			° ′ ″	Links.	Links.	Links.	Links.	Links.	Links.	Links.	Links.	Links.	
	II.	S. E.	160 12 30	409·8	...	385·6	138·8	...	...	385·6	138·8	...	...
	III.	S. E.	140 39 15	572·6	...	442·8	363·0	...	...	828·4	501·8	...	...
	IV.	E. N.	83 13 00	1594·0	188·3	...	1582·8	...	...	640·1	2084·6	...	...
	V.	N. E.	2 58 45	1442·7	1437·4	...	74·8	...	797·3	...	2159·4	...	...
	XXIX.	W. N.	101 49 15	1401·1	287·0	...	...	1371·4	1084·3	...	788·0	...	...
	I.	S. W.	35 49 30	1337·0	...	1084·0	...	782·6	0·3	...	5·4	...	Error 5·5

The above table shews, upon inspection, all the conditions and details of each point, and is a complete record of the necessary facts pertaining to each bearing and distance. Suppose, for instance, that a direct bearing or distance has been obtained from I. to V., the northing and easting opposite to V. in the total traverse portion of the table, being two sides of a right-angled triangle, the calculated bearing and distance of the hypotemuse may readily be compared with the known quantities. In a case where the bearing alone is given, the check is only partial, but where the distance is added it becomes complete. The reason of this is obvious. Referring back to Fig. 46, p. 112, a check bearing taken from I.V., as indicated by the dotted line, would bear the same as I. to x. or y., so that their departure from the correct position V. could not be determined by a bearing from I.; but if the distance I.V. is measured, x. and y. are at once proved out of position, and the error distances x.V. and V.y. obtainable. A check such as that described is the best for any piece of work, and assuming that the given distance I.V. has been taken (through the triangulation, from the original base line), it is entirely independent of all ordinary traversed work, so that a comparison with it, and the product of the meridian and perpendiculars of I. III., V., and V., XXIX., I., is the most sifting and severe test that can be applied to a surveyor's work. Another kind of check is shewn on the same Fig., viz., by means of an observed bearing III. to V., the accuracy of the chainage may be tested by subtracting like and adding unlike signs in the total traverse column of the two points: thus—

III. = 824·8 S. and 501·8 E.
V. = 797·3 N. and 2159·4 E.
1625·7 1657·6

By taking these quantities to represent either the northing and easting from III. to V., or the southing and westing from V. to III., the bearing corresponding to the observed one may be calculated therefrom, and to some extent used as a check upon the distance (see Art. VI., p. 79). Checks like these are generally handy for locating an error, and as they have to be taken at any rate for the sake of the instrument work, they entail almost no extra labour. The results of these checks should be inserted in the remarks column of the traverse sheet, opposite the point to which they refer. Before reducing the traverse lines on the Plan, it is obvious that those which had to be obtained from measured bases must be calculated; for instance, the distance *F.*XXXV. and XXXV.*C.*, and also the bearing of *F.*XXXV. must be ascertained; the latter is calculated by taking *F.*XXXV. as a side of the triangle *F.*XXXV.*A*, of which the two sides *F.A.* and *A.*XXXV. are given, and also the contained angle *A*. From this the other two required angles are calculated; also the bearing and the side *F.*XXXV. Distance XXXV.*C.* may then be obtained by making use of the triangle constructed for that purpose, viz., XXXVI. XXXV.*C.*, checking the result by re-calculating it from the angle *F.C.*XXXV. Both results should be inscribed upon the plan, as shewn, each being written upon the line and within the triangle from which it has been calculated. In the present case there is no difference, but the repetition of the figures is meant to indicate this fact. The following are the calculations in detail.

Angles *F.* XXXV.	Length *C*—XXXV.
° ′ ″	
180 00 00	° ′ ″
− 63 00 53 = contained Angle *A*.	sin. 47 30 46 = 9·8677200
	,, 50 04 56 = 9·8847832
116 59 07	489·3 = 2·6895752
÷ 2	
58 29 33·5	12·5743584
	12·7066384 = 508·9

2865·7 + 1794 = log. 4659·7 = 3·6683580
2865·7 − 1794 = ,, 1071·7 = 3·0300732
tan. 58° 29′ 33″ = 10·2125531
13·2426263
tan. 9·5742683 = 20° 33′ 58″

° ″ ″
58 29 33·5
− 20 33 58
37 55 34·5 = Angle *F.*

° ′ ″
58 29 33·5
+ 20 33 59
79 03 32·5 = XXXV.

° ′ ″
63 00 53
37 55 34·5
79 03 32·5
180 00 00

° ′ ″
sin. 79 03 32 9·9920323
,, 63 00 53 9·9499452
log. 2865·7 3·4572307
13·4071759
3·4151457 = 2601·0 Ans.

Bearing *F.* XXXV.

° ′ ″
Bearing *F.A.* = 133 02 37
− Angle *F.* = 37 55 34
95 07 03

The meridian and perpendicular of these lines are afterwards taken out in the usual way, set down, and tabulated in the same form as ordinary traverse lines; a note however, is inserted in the remarks column to shew that they are calculated lines.

The following table (p. 122) shews the single traverse results of the meridian and perpendicular of each interior line upon the plan; but as space will not admit of the traverse of each enclosure being taken out, this table is merely inserted for the convenience of the draughtsman.

As each "trig" station must be supposed to be thoroughly accurate with respect to position from the datum or initial point, a new departure is always taken when a connection therewith is effected, for example: in traversing from *D.* to *H.* and *G.*, thence back to *D.* by the road line, a connection has been made with *H.* from VIII., giving a check at that point. For reasons

TABLE No. VI.

Interior Lines—Traverse of each Distance.

No. of Station.	Cardinal Direction.	True Bearing.			Measured Distance.	On Meridian.		On Perpendicular.	
		°	′	″		N.	S.	E.	W.
VIII.-XXX.	W.N.	270	32	00	2195·0	20·4	...	...	2194·9
XXIX.-XXX.	N.W.	325	30	00	1292·0	1064·8	...	...	731·8
I.-XXIX.	N.E.	35	49	30	1337·0	1084·0	...	782·6	...
XXIX.-V.	E.S.	101	49	15	1401·1	...	287·0	1371·4	...
IX.-XXXIII.	W.N.	271	09	30	901·2	18·2	...	...	901·0
XXXIII.-XXXI.	W.N.	271	09	30	1448·0	29·3	...	...	1447·7
XXXI.-XXXIV.	W.N.	271	09	30	1555·6	31·4	...	...	1555·3
XXXIV.-XXXV.	W.N.	271	09	30	489·3	9·9	...	...	489·2
XXXI.-XXX.	S.E.	179	56	30	1076·5	...	1076·5	1·1	...
XXX.-XXXVII.	S.W.	212	40	00	1366·1	...	1150·0	...	737·3
XXXVII.-XXXVIII.	S.W.	214	59	00	859·0	...	703·8	...	492·5
XXXI.-XXXII.	N.W.	359	56	30	1190·1	1190·1	...	...	1·2
XXXII.-G.	N.W.	326	25	15	750·0	624·8	...	...	414·8
XIII.-XXXIII.	S.E.	166	32	15	1916·7	...	1864·0	446·2	...
G.-XV.	N.E.	7	46	45	1812·0	1795·3	...	245·3	...
XX.-XXXVI.	E.N.	88	29	45	1282·4	33·7	...	1281·9	...
XXXVI.-A.	E.N.	88	29	45	1246·0	32·7	...	1245·5	...
A.-XXXV.	N.E.	16	03	30	1794·0	1724·0	...	496·2	...
A.-XXVI.	S.E.	170	18	45	1621·8	...	1598·7	272·9	...
XXXVI.-XXIII.	S.W.	181	59	00	1493·0	...	1492·1	...	51·7
F.-XXXV.	E.S.	95	07	03	2601·0	...	232·0	2590·6	...
XXXV.-C.	N.E.	8	45	00	508·9	503·0	...	77·4	...

to be afterwards explained, it is requisite that all connection lines such as this should be chained at least twice. In order to effect a new departure from *H.*, in reducing the portion which lies towards *G*, to the total traverse, the meridian and perpendicular of the triangulation distance *D.H.* must be taken as the starting quantity (see table, Art V. p. 69). So that although a fresh start is made from *H.*, it may really be considered a new departure from "trig" *D.*, thereby getting rid of all error contracted or accumulated in the traverse between *D.* and *H.* Thus upon reaching *G.* another departure is taken, and so on, at the various fixed triangulation points in the survey.

It is not absolutely essential that Trig stations only should be used to connect to; any point may be used which has been fixed by a series of triangulation observations, where no Trig is available, provided the triangle is well-conditioned. A point thus formed is generally termed an auxiliary Trig, and in districts where Trig stations are wide apart or hidden from the view of an important traverse, the necessity of fixing such points is by no means uncommon, as at most, each mile of traverse should be connected to the datum Trig in some way or other by bearing and distance. In a circular traverse, and even after making the connections, some surveyors are apt to discard the application of the new departure, finding that the traverse closes better upon *itself.* Such procedure is entirely wrong, because the small error obtained is really caused by the existence of a greater and steadily increasing error from start to finish, which affects the area and any neighbouring traverse joined thereto. In the traverse sheet of the boundary traverse of the Plan (Table No. VIII.), it will be seen that if the connections had been left out altogether, and the work closed upon itself, the total error indicated would only be 0·9 South and 2·3 East, whereas, through the effecting of the connections, greater errors are shewn between points. Notice also, that if each of the errors in the traverse sheet are tabulated with respect to their cardinal directions, the sums will cancel each other, so as almost to agree with the result of the traverse as closed upon itself; the slight difference being attributable to defective triangulation, thus:—

TABLE No. VII.

TRAVERSE.	*N.*	*S.*	*E.*	*W.*
D. to *H.*	1·40	...	2·20	...
H. to *G.*	...	3·50	3·30	...
G. to *F.*	4·00	...	7·20	...
F. to *E.*	...	0·50	1·30	...
E. to *D.*	...	2·30	...	11·10
	5·40	6·30	14·00	11·10
		5·40	11·10	
		0·90	2·90	
Error	"Self Closing"	0·90	2·30	
	Diff. =	0·00	0·60	

Placed upon the next page is the traverse table of the Plan survey, as referred to in this and the following Art:—

The series from *D.* onwards to *H.* is closed back again to *D.*, thus completing the circuit by tabulating the meridian and perpendicular of *H.D.* as an ordinary traverse line, the quantities having been obtained from the table (Art V., p. 69). The closing error is then made to appear in the remarks column, opposite *D.* In starting onwards upon the next series, the correct position of *H.* is set down to begin with in the total traverse column, and the connection line to VIII. tabulated in its correct form. When closing in, however, at *G.*, the error may readily be found, not in tabulating the Trig distance in the form of an vers

	Area.		
	Products.		Result.
	N.	S.	
D.			
I.	...	10·79231	
II.	...	39·55977	
III.	...	47·64566	
IV.	23·90977	...	
V.	206·29570	...	
VI.	·27998	...	
VII.	101·18777	...	
III.	97·32774	...	
H.	...	...	
D.	...	...	
H.	...	...	
III.	...	...	1002·93909
IX.	154·99359	...	228·65097
X.	90·06500	...	
XI.	176·27458	...	2 | 774·28812
XII.	49·51031	...	
III.	...	20·88150	Total of Traverse 387·14406
G.	...	2·76931	,, Offsets 2·37498
D.	...	...	389·51904
G.	...	...	4
IV.	38·95366	...	
XV.	5·81209	...	2·07616
VI.	...	14·58272	40
VII.		14·72652	
III.	4·64274	...	3·04640
IX.	...	10·71587	
F.	...	16·09424	
D.	...	...	
F.	...	...	*Ac. Ro. Po.*
XX.	...	10·26899	389 - 2 - 03
XI.	...	·56447	
E.	...	...	
D.	...	...	
E.	...	...	
XI.	...	...	
XII.	...	·93442	
III.	2·04734	...	
XIV	...	7·06679	
XV	...	7·98225	
VI	21·03334	...	
VII	30·60578	...	
III	...	9·50627	
D	...	14·55988	
	1002·93909	228·65097	

ordinary closing line (as in the first series, which occupies more time), but on turning back to the triangulation table, a comparison with the meridian and perpendicular of *G.* will at once shew the error. As the result is the same by both methods, it is really of little consequence whether the traversed circuit is tabulated in detail, or the result of the comparison indicated by simply noting down the error upon the margin.

Although it is not justifiable to alter the actual observation and chainage results after these have been executed upon the ground a second time, the necessity of doing away with the error by distribution is none the less apparent. The first portion of the traverse table, as far as its remarks column, must therefore remain intact as a true and faithful record of the field operations, the amount of error, and where it was contracted and discovered. The method usually adopted in distribution, is to divide the error by 2, subtracting and adding the result in proportion to each quantity contained in the meridian and perpendicular columns of the same traverse, leaving the connection lines entirely out of account, because of their extra treatment upon the field. The error proportion thus obtained must necessarily be added to the figures in the column shewing the smaller summation, and subtracted from those of the other. For example, referring to the traverse table of the Plan, there is an error noted 1·4 North in the first series, indicating that the northings exceed the southings to that amount, but (leaving out the connection line) as all the error has been made between I. and VIII., 1·4 falls to be distributed between *D.* and VIII., therefore half the error=0·7. must be deducted from the single traverse northings and added to the similar southings in the percentage, or proportion which each separate quantity in the northing or southing column bears to their sum in the series. The error of 2·2 in the easting and westing is treated in the same way, so that each pair of the four columns must exactly balance.

Each separate series having been thus treated, the results are tabulated in the columns marked upon the traverse sheet as "corrected single traverse." The "corrected double traverse" is then extended in the manner shewn (p. 117), finishing off by inserting its remarks as an indication that the corrections have been properly carried out. In this form the corrected results may be used for the calculation of the area, or the plotting of the plan.

IX. THE AREA.

As surveys in general are made in order to obtain the area of the ground over which they extend, the computation, or as it is commonly called, "taking out" of this area is an operation of considerable importance. While there are different systems in practice applied to the field work of surveys, there are also different methods of taking out areas.

The most common, and indeed almost universal system is, to calculate the area from the plan, by laying the enclosure off into squares or triangles, with pencil and straight-edge, taking the necessary measurements by means of the scale used in plotting. This is the method by which surveyors ascertain the contents, who plot their surveys by means of the protractor (Art. V., page 64), and do not reduce the traverse to the meridian and perpendicular. But it necessarily follows that the accuracy of the area thus obtained not only depends upon the quality of the field work, but also upon the exactitude with which the plotting has been done. Again, the paper upon which a plan is drawn is very liable to expand and contract, and supposing the plot to have been made some time before the area is taken out, the condition of the paper with this respect is certain to have changed. It is true that the amount of change in the paper may be roughly ascertained by having previously pricked off a scale thereon, whereby an allowance may be made for future readings; but after all, this estimate is only an approximate quantity, not giving the difference to the decimal of a link, probably not even to a link, so that although this precaution might serve to indicate that a change in the normal condition of the paper had taken place, its amount would not be mathematically reliable. Notice also, that in so taking out an area by scaling, it is not the actual chainage distances that are used in the calculations, but distances obtained by scaling a series of lines upon a plot, and with a scale which may be far from accurate.—The result being merely the superficial area of that part of the paper enclosed by lines supposed to represent the boundary of the real enclosure. In order to arrive at some sort of idea as to the effect of shrinkage errors, the following experiments were made:—

1. An area to the scale of 1 chain = 1 inch was scaled, and found to give 12ac. 1ro. 01pls.; whereas by the meridian and perpendicular system 12ac. 1ro. 09pls. was the result; shewing an error of 8 poles.
2. An area to the scale of 1½ chains = 1 inch was scaled, and found to give 20ac. 3ro. 13pls.; whereas by the meridian and perpendicular system 20ac. 3ro. 17pls. was the result; shewing an error of 4 poles.
3. An area to the scale of 3 chains = 1 inch was scaled, and found to give 13ac. 1ro. 18pls.; whereas by the meridian and perpendicular system 13ac. 1ro. 23pls. was the result; shewing an error of 5 poles.

In every case, the scaled area is less than the actual one, indicating shrinkage of the paper between the plotting and the taking out of the area. It may be further stated that these examples had the advantage of being plotted by the meridian and perpendicular method, and during the whole work the plans were treated with ordinary care. Upon comparing cases I. and III., it is seen, that although the total areas are similar, the discrepancies are greatly different, which can only be accounted for by the dissimilarity of scales. This evidence shews it is not correct to suppose that the larger the scale the greater the accuracy of the plan, and points out, that any good result anticipated through the use of a large scale is entirely destroyed (so far as the area is concerned) by the increased shrinkage surface. Although scaling is not so correct as the meridian and perpendicular system, it is handy as a means of checking more intricate work, and may be used for laying off probable quantities or trial areas.

In order to apply the scale to its best advantage for the calculation of an area, take the following as an example. Suppose Fig. 47 to represent the plot of a field, the area of which is thus to be ascertained. Proceed to lay down a trapezium and series of triangles with a fine pointed pencil. Let these be represented in the Fig. by the dotted lines. Draw in the diagonals *a.d.* and *e.b.* Scale the length of one of the diagonals *e.b.*, and from the same diagonal carefully measure the two perpendiculars *a.f.* and *g.d.*; then the length of the

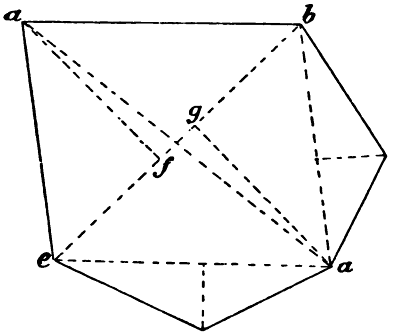

Fig. 47.

diagonal *e.b.*, multiplied by the mean of *f.a.* and *g.d.*, should equal the area of the trapezium *a.b.d.e.* Proceed in the same manner with the other diagonal *a.d.*, the mean of the two total results may be accepted as the true area of the trapezium. Then calculate the remaining triangles by multiplying their bases by half their perpendiculars. The total area of the whole may be re-checked by rubbing out the penciling, and laying off the figure again in a different way. Care must be taken to make the pencil lines perfectly straight and fine, and the exact points of intersection should be defined by a small needle prick-mark.

There are several ways of taking out areas by the meridian and perpendicular system, but it will suffice to give only two examples.

In Fig. 48, let *A B.C.D.* be a plot of an enclosure, the area of which is to be calculated. Through the most westerly point,

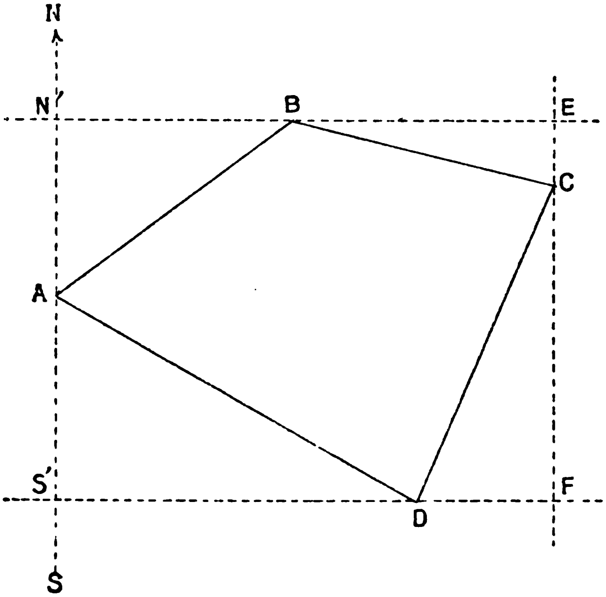

Fig. 48.

A., draw the meridian line *N.S.*, and draw a similar line through *C.*, the most easterly point; lastly, the perpendicular lines *N′.E.* and *S′.F.* through the most northerly and southerly points. By referring to the corrected total traverse column the distances *N.S.* or *E.F.* and *N.E.* or *S.F.* may be found, and by multiplying the two quantities together, the area *N.E.F.S.* will be the result. From this area, and by referring to the corrected single traverse columns, subtract the triangle *A.N.B.* by multiplying the northing of line *A.B.* by half its easting, or *vice versa.* In a similar manner, by subtracting the other three triangles, the remainder will be the required contents of the enclosure *A.B.C.D.* This system is manifestly superior to the scaling method, because the actual chainage distances are those used in its computation, and bad draughtsmanship, defective scaling, or shrinkage in the paper cannot affect it. The only objection to this system is, that in applying it to a plot where the boundary is irregular, while generally running in the same direction, it creates a number of small figures which are liable to confuse the calculator; also it is necessary to have the plot drawn before the area can be taken out. In some figures, however, this system may be found simpler and quite as expeditious as any other.

The following, as set forth in the traverse sheet of the Plan, is recommended as being the most convenient, accurate, and easily applied manner of taking out trigonometrical survey areas. Its working is as follows:—

Ascertain from the "corrected total traverse" columns the most westerly point in the circuit, that is to say, not including connection lines, but taking the traverse in one continuous series all round. Take this (peg XXI. upon Plan) as the starting or initial point, and from it proceed to make a fresh "perpendicular" column of "totals," inserting each quantity upon its own line and within its proper column,— the initial point must of course be represented by zero. Then sum up each pair of these quantities, all of which are eastings, writing down the results in the column set apart for them, and in a line with the lower of each added quantity. Proceed next to transfer the "corrected single traverses" north and south into one column, adjoining the one already treated; then multiply the quantities in the "sums of each pair" column by those immediately opposite to them in the "north and south" columns, inserting the answers into the north column of products, when the multiplier is north, but into the south when the multiplier is south—half the difference of the north and south products will be the area required.

As this is only the contents of the ground enclosed by the traverse, the field-book has to be referred to, so that the offsets taken between the traverse and the boundary may be read off, enabling the surveyor to add all parcels of land lying without the traverse lines, and deduct these which lie within.

Thus the whole area, as taken out by this method, has been made to depend from first to last upon actual ascertained *field facts*, and may be accepted with all assurance of precision.

X. THE PLAN.

The plotting and drawing of a plan by the meridian and perpendicular system is carried out in conformity with mathematical exactitude; and is, like all other operations already described, based upon calculation as opposed to mechanical agency. As explained in Art. V. page 68, the trig points are first laid upon the plan, the initial point being shewn in a more prominent manner than the others. Through this point a meridian line is drawn, likewise also the perpendicular, both of which should be finely executed in blue ink—great care being taken to get them precisely perpendicular to each other. Each

Trig point should then have its reference letter appended; and the bearings and distances should be inscribed upon all the lines of the plan as shewn. It is advisable to draw on the scale at an early stage of the work in order to indicate any change in the condition of the paper. Proceed to plot the traverse by pricking off the "corrected totals" in the table, either upon the meridian or the perpendicular line; and through these prick marks rule pencil lines at right angles, that is to say, parallel to the first meridian or perpendicular as the case may be. Upon each pencil line prick off the distance from the first meridian or perpendicular, as also set forth in the "corrected totals" columns, the lines joining each point thus pricked off will represent the traverse. Check all the distances with the scale, in order to see that they measure the same as given in the "measured distance" column. Mark the number of each station in neat Roman figures, and making use of bow compasses draw a small red circle round each prick mark; ink the chain-lines in with red, marking thereon the bearing and distances. Proceed in the same way to lay down every point of the traverse; and where it is found that the scale does not shew the exact chainage of a line, carefully repeat the laying down of the meridians and perpendiculars of the adjoining lines, until the mistake is discovered and rectified.

The usual way of shewing a distinction between the different classes of lines and figures is to shew them in different colours. The following is the manner almost universally adopted, thus:—

Chained lines in red.
Calculated lines in black.
Observed, but not calculated, lines in blue.
Chained distances in red.
Calculated distances or bearings in black.
Observed bearings in blue.
Trig stations by a large and small circle in red.

Ordinary stations of present survey by small circle in red.

Stations of former survey by small circle in black.

Having thus drawn and inked in the lines as shewn on Plan including check-lines, and every piece of information which the surveyor can supply, the field-book is referred to, and the boundary offsets laid down therefrom, also every detail such as houses, roads, streams, division fences, etc. The whole neatly drawn in, enables the plan not only to become a true delineation or map of the ground, but also a record to set forth the manner in which it was surveyed, and the amount of accuracy attained in the survey.

It may be considered unnecessary to enter into further details as to the finishing of the plan, because these may be left to the taste and experience of the draughtsman himself. Although it is evident that the application of the meridian and perpendicular system of surveying is more intricate than the popular method, by a little extra study and care, so as to master the routine, it will be found to repay the surveyor who adopts it, by proving more interesting, scientific, and accurate. Surveyors in this country might advantageously work the boundary traverse and area by this system, using the scale in taking out the interior enclosure areas, distributing proportionately any errors throughout. The best and safest way, however, is entirely to conform with the system, feeling assured that its attainments will more than repay the labour.

JAMES MUIR, PRINTER, EDINBURGH.

ERRATA.

Page 1, 8th line from foot, *for* “66 + 12,” *read* “66 × 12.”
,, 5, 5th line from top, *delete* “not.”
,, 28, 12th line from foot, *for* “29 × 30,” *read* “29 + 30.”
,, 64, Centre circles should be reduced in diameter, as per text.
,, 71, 7th line from foot, *for* “343° 12′ 40″,” *read* “310° 23′ 55″.”
,, 74, 6th line from top, *for* “Fig 26” *read* “Fig. 24”
,, 104, 4th line from top, *for* “following,” *read* “preceding.”
,, 112, 3rd line from top, *for* “north,” *read* “south.”

40 CHA